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A CORE CURRICULUM IN MATHEMATICS FOR THE EUROPEAN ENGINEER

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PREFACE - By the Vice-President of SEFI

Mathematics is not only the queen but also the handmaiden of the sciences. In engineering the assistance of this servant is indispensable, no technical discipline can afford to ignore her. In a sense this has always been true, but it is becoming ever more so as the complexity of engineering tasks grows and abstract information processes start playing essential parts in technology. Edward E David, President of Exxon Research & Development, stated in a report to the National Science Foundation of the USA: 'Too few people recognise that the high technology so celebrated today is essentially a mathematical technology'. Everywhere in the world mathematicians devote much time and energy to educating engineering students in the time-proven insights, and modern methods, of mathematics.

The Société Européenne pour la Formation des Ingenieurs, SEFI, has brought together many of the leading teachers of mathematics to engineers in an active working group. This SEFI working group on Mathematics has now produced a report: 'A Core Curriculum in Mathematics for the European Engineer'. This Core Curriculum, the fruit of hard work of many experts, reflects the considered opinion of leaders in the field of mathematical education for engineering. So, although this curriculum cannot legally be enforced, every professor and every school is completely safe in following it. There has been such a report in the past: 'Mathematical Education of Engineers', published by the OECD in 1965, but the rapid changes since that time, notably in computing power, require a new version.

In the present SEFI document the report is preceded by the address with which it was presented by former SEFI President, Professor P E Nüesch of the Swiss Federal Institute of Technology, Lausanne. This address highlights the historical background and educational environment of mathematics for engineers, thus underlining the great importance of the Core Curriculum report.

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PREFACE - By the Vice-President of SEMI

Microelectronics is not only the power but also the backbone of the economy. In carrying the momentum of the economy is microelectronics, an electrical discipline and effort to improve it. In a sense this has always been true, but it is becoming ever more so as the complexity of engineering tasks grows and electrical disciplines become more highly integrated parts in technology. Edward E. Lewis, President of Union Research & Development, stated in a report to the National Science Foundation of the USA: "The two people everyone that the high technology as practiced today is essentially a microelectronic technology." Everywhere in the world microelectronics drives much new and energy in electronic engineering wherever in the three power fields, and modern methods of microelectronics.

The Institute of Electrical and Electronics Engineers (IEEE) has always been active in the field of the leading leaders of microelectronics in engineering in an active working group. The IEEE working group on microelectronics has now produced a report, "A Case for Microelectronics in Education." The Case Committee, the first of kind of any report, states for microelectronics in education the field of microelectronics education for engineering. It is through this committee report that we received every graduate and every school a microelectronics education. This has been such a report in the past. Microelectronics education of engineers, particularly in the USA in 1984, has the rapid change since that time, notably in computer power, require a new effort.

In the present SEMI document the report is presented by the address with which it was prepared by former SEMI President, Professor F. E. Lewis of the Texas A&M University of Technology, Lubbock. This address highlights the historical background and educational development of microelectronics for engineers, from including the past activities of the Case Committee report.

Professor F. E. Lewis
Professor of Technology
Lubbock
Texas

A WELCOME ADDRESS - from the Engineering Community

It has been maintained that the first half of our century has been scientifically dominated by the great developments of physics, but it is my firm belief that the second half is going to be considered as the age of the development of computing machines. From the 1500 electronic tubes of Colossus during the Second World War to today's PCs, computer networks, and bit parallel machines, the role of computers has steadily increased. They are now involved in every aspect of science and engineering. Problems once too complex, even to be approached, are now routinely solved.

If one were to ask in the streets of any European city as to why this has happened, most people would point to the outstanding achievements of micro-electronics, or, as Ralph Gomory once said, to the fact that 'a small bit is just as good as a large bit!'. This is however only one side of the coin, the other is entirely mathematical. Computers solve problems using algorithms, either numerical ones, like those common in the classical branches of civil or mechanical engineering, or non-numerical ones, like those of management science and operations research. Not only is this true, but the development of computers themselves rests upon very deep mathematical roots. It was in order to solve Hilbert's Entscheidungsproblem that Alan Turing conceived his abstract machine which would have become the most widely used general model of computation. In addition the applicability of even the most theoretical aspects of mathematics is ever increasing, helped by the channel of informatics. For example, consider the relevance of that most 'pure' of all branches of mathematics, the Theory of Numbers, in modern cryptology.

As a consequence the mathematical Core Curriculum, of students in engineering, needs continuous updating and the efforts of the SEFI Working Group in Mathematics cannot be welcomed enough. These efforts have produced a remarkably well thought out and balanced list of topics, fundamental and optional, which is going to constitute a reference for engineering schools all over Europe. I consider this function an essential one. The resistance of conservative academic environments to the necessary periodic updating of the mathematical core of curricula is well known. It has often constituted a real problem in European engineering faculties. This Core Curriculum proposed by SEFI, with its qualified presence of extended parts in Discrete Mathematics, and in Probability and Statistics, comes as a relief to the many of us, who will have from now on valuable help in their efforts to improve the mathematical preparation offered in European engineering schools.

Professor F Maffioli
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A WELL-COME ADDRESS - from the Engineering Community

In the first instance, the fact that we are here today is a tribute to the efforts of the Government and the industry in providing us with the facilities and resources necessary for the development of our profession. It is a privilege to be able to meet with you today and to discuss the many issues that are of concern to us all.

It is a pleasure to be able to meet with you today and to discuss the many issues that are of concern to us all. The Engineering Council has been set up to represent the interests of the engineering community and to work towards the improvement of the standards of our profession. We are pleased to be able to meet with you today and to discuss the many issues that are of concern to us all.

As a member of the engineering community, it is our duty to ensure that we are always at the forefront of our profession. We must continue to work hard and to improve our skills and knowledge. We must also ensure that we are always at the forefront of our profession. We must continue to work hard and to improve our skills and knowledge.

Professor J. Smith
Department of Engineering
University of London
1971

ADDRESS - To the 1991 SEFI Annual Conference in Marseilles by the 1990-91 President of SEFI

WHY A CORE CURRICULUM IN MATHEMATICS FOR ENGINEERS?

The half-life of knowledge an engineer acquires, in undergraduate instruction, is estimated to be somewhere between five and ten years, depending on the subject, orientation, etc. Of course, knowledge in mathematics has a much longer half-life. We, from the community of mathematicians, want to believe that there is no exponential decrease in the value of mathematical instruction. Unfortunately, it is not the value that decreases exponentially, it is the mastery of mathematical skills that decays to zero once the engineer has left school. In times like today, when the cry 'back to basics' is heard, the need for mathematics instruction is not controversial due to its intrinsic value. Controversy starts only if one has to answer questions such as:

What to teach?

How much to teach?

By whom is the teaching to be done? etc

The OECD report in 1965, *Mathematical Education of Engineers*, [1], dealt with this problem a quarter of a century ago. In the meantime mathematics teaching in engineering schools has changed drastically, due mainly to the ready availability of computers, ranging from pocket to lap top to super-computers, and also to the abundance of good or even excellent textbooks. This is particularly true for English written texts and less so for other languages. What has changed as well, in these past twenty-five years, is the profile or preparation industry demands of a scientific engineer. Industry demands engineers better educated mathematically, and it demands an ever growing number of them. We all know of the Finniston report, published in 1980, [2], with its strong emphasis on engineering practice. Today it is felt that his report does not meet the required profile of the scientific engineer.

The Mathematics Working Group of SEFI started the work which resulted in this report ten years ago, around the time of the publication of the Finniston report. It drew on expert advice from all countries in Western Europe. Of course an ambitious project like this could not expect to have an easy ride. There were controversies and heated discussions right up to the last Working Group seminar in Balatonfüred, Hungary, in the spring of 1991. Nevertheless it may be said that, with diplomatic skill and the power of conviction, a consensus on the final version has been achieved. The report exists in three languages, English, French and German, to guarantee a wide circulation. It is the identical report in each case. We deliberately don't stress the cultural and historical differences that are reflected in the mathematics teaching associated with these language groups. The Mathematics Working Group of SEFI is proud to present its report as some sort of silver jubilee of the OECD report. A comparison to this famous predecessor of twenty-five years ago shows that our Core Curriculum (CC) does not so much modify the OECD courses as change their emphasis. In no way do we claim that the OECD findings are obsolete. We hope that our report reflects the major changes that have

occurred since 1965. Mathematics for Computation becomes Informatics or Computer Science and is so basic a subject, not only to mathematics, that it is largely taken out of the CC. Algebra and Analysis are separated, and Numerical Analysis, which twenty-five years ago was listed under Mathematics for Computation, is now an integral part of Analysis instruction. Where this report differs from the OECD report is in Discrete Mathematics. In the OECD report it is not mentioned. What it calls mathematics for computation has now been replaced by Discrete Mathematics and Operations Research, respectively.

The four components of the curriculum presented in this document are adaptations of four different reports:

Analysis by Michael Barry

Linear Algebra by Peter Nüesch

Probability and Statistics by Lennart Råde and Romano Scozzafava

Discrete Mathematics by Nigel Steele and Robert Critchley

There is much more in these individual reports than meets the eye. Much of the material, of these sub-reports, had to be left out in order to reduce the CC to a readable size. Michael Barry and Nigel Steele who compiled the final version must have felt like the editors of a film. Much footage, however valuable, needed to be thrown out in the final cutting process. I am sure that any of the six authors mentioned would be glad to make their report available.

The report makes no distinction between mathematics instruction in universities and in polytechnics, this is intentional. We believe that it is not primarily a difference in content but rather a difference in level in the various schools, although an adequate number of course hours in some places makes it difficult to teach all we think is needed. It is hoped that our Core Curriculum answers only the one very essential question: what should be the content of mathematics courses for engineers? Whilst it is of the utmost importance to educate the engineer at the end of this century, it is not the only problem we face. There are often problems beyond the influence of an individual instructor or even an entire mathematics department. The main one is not what to teach or how to teach or even how much should be taught. Very often the good, modern, extremely useful mathematics taught at the beginning of the degree is not taken up by engineering professors in their respective courses. Thus in the minds of the students mathematics is used to select negatively. They want to get it over as painlessly and as quickly as possible. How can we blame students for this attitude when they seldom use in subsequent courses what they learned in their mathematics course and when mathematics instruction is confined to the first half of the study period. The engineering side may prefer to have mathematics courses taught by engineers to make them more motivating for students. This has to be resisted. Experience shows that mathematics courses are best taught by mathematicians who are aware that their audience is composed of future engineers. It is even better if the mathematicians have a research interest in an engineering subject. In addition mathematics should be taught throughout a three or four year course so that it is fully identified with the whole of the engineering degree curriculum.

I repeat here the principles which were elaborated by the Working Group in Dublin and Budapest and which go with this Core Curriculum.

1. Mathematics courses for engineers are best taught by mathematicians.
2. Mathematics should accompany a student from entry to degree.
3. The use of computers is an integral part of mathematics courses.
4. Mathematics courses should be taught with a strong emphasis on applications and projects.
5. Concentrated mini courses in mathematics should be avoided.

In the working group discussion it was concluded that a general level of 15-20% of total course time should be devoted to the study of mathematics on all engineering degree courses. With this scale of time understood, mathematicians can advise upon and provide the most appropriate content of each course for specific engineering disciplines. The Core Curriculum will help both mathematicians and engineers to do this.

Professor P E Nüesch
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REFERENCES

- 1 MATHEMATICAL EDUCATION OF ENGINEERS, 1965, OECD Report.
- 2 Engineering our Future, Report of the Committee of Inquiry into the Engineering Profession, 1980, HMSO, Cmnd 7794.

I would like to express my appreciation to the members of the Working Group in 1980 and to the members of the Committee on the Study of Mathematics.

1. Mathematics courses for engineers and scientists should be designed to provide a solid foundation in the subject.
2. The role of computers in an important part of mathematics courses.
3. Mathematics courses should be taught with a strong emphasis on applications and projects.
4. Computerized math courses in mathematics should be avoided.

In the working group discussion it was suggested that a general level of 15-20% of total course time should be devoted to the study of mathematics in an engineering degree program. With the state of the art in mathematics, communications can be improved and the most appropriate content of such courses for specific engineering disciplines. The Committee will help with mathematics and engineers to do this.

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THE COMMITTEE

1. MATHEMATICAL EDUCATION OF ENGINEERS, 1980, GOOD REPORT
2. Engineering and Science, Report of the Committee on the Study of Mathematics
Washington, 1980, NRC, QED 174

1 FOREWORD

Engineering education is addressed by a number of institutions notably SEFI (The European Society for Engineering Education) which is a non- governmental organisation. Founded in 1973, its objectives are to discuss systems of engineering education and training throughout Europe, analyse and study problems created by the rapid developments in science and technology, and contribute to the development and improvement of the education and training of the engineer by adapting them to the needs of the contemporary industrial world. It has a permanent secretariat in Brussels and continues to expand its role. Many of the activities of SEFI are conducted through its various working groups. One such group is the Mathematics Working Group (SEFI-MWG). Founded in 1982 under the co-chairmanship of Professor D J G JAMES (Coventry University, England) and Professor K SPIES (University of Kassel, Germany), and succeeded in 1990 by Chairman Professor L RÅDE (Chalmers University Gothenburg, SWEDEN) and Vice-Chairman Dr F H SIMONS (University of Eindhoven, NETHERLANDS), the aims of the SEFI-MWG are:

- to provide a forum for the exchange of views and ideas among those interested in engineering mathematics
- to promote a fuller understanding of the role of mathematics in the engineering curriculum, and its relevance to industrial needs
- to foster cooperation in the development of courses and support material
- to recognise and promote the role of mathematics in the continuing education of engineers in collaboration with industry

To fulfil these aims and maintain the essential international participation the SEFI-MWG held seminars between 1984 and 1988 in different European countries, namely:

- 1st Seminar - University of Kassel, Kassel, FR Germany, March 1984
- 2nd Seminar - Engineering Academy of Denmark, Lyngby, Denmark, March 1985
- 3rd Seminar - Polytechnic of Turin, Italy, March 1986
- 4th Seminar - Chalmers University of Technology, Gothenburg, Sweden, April 1987
- 5th Seminar - Plymouth Polytechnic and Royal Naval Engineering College, Manadon, Plymouth, England, March 1988

The earlier seminars concentrated heavily on exchanging and gathering information and interpreting the SEFI-MWG's aims as looking into development and innovation, the impact of computers and the mathematical tool-kit of engineers. By the 4th Seminar these had evolved into a closer definition of those self same aims. The use of computers, and particularly micro-computers, both in and out of the classroom, was providing lecturers and students alike with a wide variety of services ranging from user-friendly

conventional computing to computer algebra and interactive video and computer enhanced learning. The principal mathematical training of professional engineers varies widely according to specialisation and country but there appears to be a general agreement that there is a need for a Core Curriculum in mathematics for engineers and that it should include:

- Analysis and Calculus
- Linear Algebra
- Discrete Mathematics
- Probability and Statistics

and that numerical methods should be integrated into the Curriculum.

Four working sub-groups were involved in framing each of the four main parts of the Core Curriculum. Considerable overlap exists between these parts and institutions may prefer a modified itemisation of the Core Curriculum elements listed. The SEFI-MWG convened a Colloquium of the working sub-groups in Kassel, FRG in November 1989 to agree the detail and emphasis of the Curriculum and the international consensus of the Colloquium resulted in this document.

2 INTRODUCTION

The proposal aims to justify, specify and itemise a Core Curriculum (CC) in mathematics that can be adapted for teaching mainstream students of engineering in institutions throughout Europe. It is intended to act as a guideline and not as a definitive syllabus recognising that every institution has its own engineer formation requirement and that certain countries such as the United Kingdom conduct three year engineering degree courses whereas others, notably Germany, Italy and France adopt a five year programme to train engineers to a professionally recognised level. The expectation as general standards, dictated by economic and peripatetic factors, are progressively applied in the wider European Community (EC), post 1992, may be that international engineering education patterns may be under pressure to refer to a common standard whilst maintaining their own individual character to offer overall educational variety. With this in mind a CC has been progressively assembled over the past five years by nearly 200 academics and engineers from eighteen countries acting under the direction of the SEFI-MWG. The CC proposal identifies four main mathematical disciplines that need separate consideration to meet emerging technological needs together with three levels of hierarchical sequence, ie assumed prerequisite knowledge, Core Curriculum, and examples of additional Core or elective course material. It is strongly emphasised that the Core itself represents the ABSOLUTE MINIMUM, of pure mathematics that should be made available to engineering students in any European institution recognising that some institutions, notably those awarding degrees of greater length, will offer a much broader Core. The proposal also addresses the engineer's requirement for the mathematics, the influence of technology in its various aspects, the educational process and the continuing education he or she is expected to undergo. It is being co-ordinated by the Editor, Dr M D J BARRY (RN Engineering College, Manadon, Plymouth, UK), the Secretary and Editor, Mr N C STEELE (Coventry University, UK) in collaboration with the SEFI-MWG Chairman, Professor L RÅDE (Chalmers University, Gothenburg, Sweden) and the Chairman of SEFI, Professor P NÜESCH (École Polytechnique Fedrale, Lausanne, Switzerland). It is intended that the Core Curriculum will be circulated to every tertiary institution throughout Europe which is a member of SEFI. Versions are available in ENGLISH, FRENCH and GERMAN so that the document is fully accessible to most European engineers and their educators.

3 THE CHANGING FACE OF MATHEMATICS IN ENGINEERING EDUCATION

3.1 THE ROLE OF MATHEMATICS

1. Throughout history mathematics and engineering have developed hand in hand, and from time to time it is appropriate to review the health and nature of this relationship. The dependence of all branches of engineering on mathematics for their description, and the rich flow of ideas and problems from engineering which have stimulated and sometimes initiated branches of mathematics, provide ample evidence of past mutual benefit. In undertaking such a review, note should be made of the conclusions of past reviews, where relevant. In particular, at a time of rapid technological change, earlier analyses of the importance of an understanding in mathematics to the practising engineer merit careful attention.
2. The O.E.C.D [1] identified a number of reasons for the importance of mathematics as a subject for the engineering curriculum. These reasons can effectively be summarised as follows:
 - a. Mathematics provides a training in rational thinking and justifies confidence in the value of such thinking.
 - b. It is the principal tool for the derivation of quantitative information about natural systems.
 - c. It is the second language of human discourse and parallels natural language by providing a means of communication for ideas, as evidenced by the contents of technical papers.
 - d. It facilitates the analysis of natural phenomena.
 - e. It is important in assisting the engineer to generalise from experience.
 - f. It trains the imagination and inquisitiveness of the student if properly taught.
 - g. It is an education for adaptation to the future.

These reasons as general principles remain valid, however the arrival of the digital computer has brought about the need for at least two further principles.

- h. Mathematics provides the language for formulating a model for computer analysis.
- i. Mathematics provides the means of understanding how a computer operates and the computing process itself, and the means of assessing the accuracy of computer output.

Items b., d., and h. relate to the development of professional communication and computation skills appropriate to an engineer. The remaining items identify mathematics as central to the intellectual formation process of the engineer.

3. In this latter connection, the pace of change in recent times has ensured that the technological aspects of engineering training will rapidly become outdated. Thus,

engineers during the course of their professional lives may expect to undergo a number of periods of technological updating, involving the understanding of new concepts as well as the mastery of new techniques. In some cases, the updating process will involve the attendance of career courses etc., but given an adequate educational base, significant continuing education can take place through personal study and experiment. For this reason, those tasked to teach engineers in various European countries have adopted a strategy aimed at providing education for change, rather than a training for life. Notwithstanding the wide variation which currently exists in the provision of continuing education and training programmes between different countries, it is vital to appreciate the importance of fluency and understanding in mathematics for the success of such programmes in engineering. Without the means of understanding modern technological publications, almost inevitably written using mathematical notation and with arguments mathematically expressed, the intended beneficiary could be reduced to the role of a passive observer in his developing field.

4. There is a strong desire for mutual recognition of engineering qualifications across national boundaries, particularly among member states of the EC. There are a number of difficulties to be surmounted before such mutual recognition will take place, not least concerning the length of degrees in different countries. There are also observable differences in the design of such degrees, especially in the importance attached to the mathematical and physical foundations of engineering. There appears, at least superficially, to be a correlation between those countries where there is a greater emphasis on the understanding of underlying concepts in engineering, and economic success in manufacturing.

3.2 NEW TECHNOLOGIES

1. The development of new technology, notably computers, is doubtless the largest single factor that has affected the redesign of a Core Curriculum following precedents such as [1]. Computer usage is now commonplace throughout the spectrum of engineering disciplines. Many professional engineers can expect to make frequent use of software packages, and at various times throughout their working lives will need to specify, write, amend and extend computer programs. The resulting systems will often be of a highly complex nature, whose output may have a perceivable, possibly even critical, effect on the daily lives of members of the community. A new discipline, software engineering, has emerged and calls have been made for the establishment of software standards and a form of software certification. With this in mind it is important that detailed consideration be given to an appropriate form of education in computing to prepare engineers for the demands that will be made upon them. In particular engineers, and notably software engineers, will need special skills in formal methods and the underlying discrete mathematics in order to validate and test software. At a different level, engineers from certain disciplines are likely to be involved in the design of computers, both large and small, and they too will require further specialised knowledge of discrete mathematics and its applications.

2. The computer is nowadays capable of contributing to the teaching activity itself, and valuable insight can be gained by the informed use of available software and courseware to enhance the presentation of material. It is also true that commercially available software packages can significantly reduce the manipulative effort, and thus the need for over-highly developed manipulative ability on the part of the practising engineer as well as the student. Emphasis is therefore even more strongly placed on the need to understand concepts, rather than to develop supreme mastery of the techniques necessary to produce sophisticated analytical solutions or designs. The arrival of the computer has required of the user a greater competence in the higher intellectual skills, in particular, mathematical representation, or modelling. The trade-off is that the engineer of the 1990s has a reduced requirement for the less interesting and more repetitive skills of his predecessor, for example, the hand evaluation of heat and stress distributions. The influence of the computer will continue to gather speed notably in the demand for discrete system modelling and may even affect hitherto unaffected parts of the curriculum. This will likely lead to a complete reappraisal of this Core Curriculum proposal after a period of about ten years. Engineers will however still need to understand continuous concepts so that they can intelligently assess the value of information which is embedded within the discrete, algebraic or numerical output provided by computers. To do this the underlying mathematical model must be understood fully together with the discretisation process so that judgements based on numerate commonsense can be made of computer output data. In particular special skills to understand the scope and limitation of increasingly powerful computer software will be needed so that sensible interpretations are made of highly sophisticated results.

4 THE EDUCATIONAL PROCESS

4.1 EDUCATIONAL OBJECTIVES

The engineer need not and possibly should not be an expert in mathematics 'per se' and his or her education and development need to reflect the present and future demands that will be made upon him or her. This means that there is no need for unnecessary mathematical rigour but special attention needs to be paid to the following abilities:

1. To interpret and be able to solve a modelled problem whose solution requires the direct and minimal application of mathematics or statistics. To know when to seek expert advice.
2. To communicate effectively, both orally and in writing, the results of analytical or statistical investigations. To understand literature that contains the direct and minimal use and application of mathematics or statistics.
3. To understand elementary mathematical models of engineering problems, for example, those whose analysis leads to an initial or boundary value problem in linear differential equations or to the analysis of data or simulation. To be able to interpret sensibly the solutions to such problems as generated by computer software packages.
4. To continue professional development at some future time by having the intellectual means of access to relevant technical literature for both formal and informal continuing education.
5. To understand how a computer works and how it processes data.

4.2 ENTRY POINTS

1. A non-homogeneous entry base to engineering degree study is becoming more common in European institutions as students with wide variations in earlier mathematical and scientific education come forward. At one time the onus was placed upon the student himself to make good his academic fitness to embark upon a degree but increasingly entry points are being adjusted to meet the wider ranging but usually lower level academic qualifications being offered by a majority of students. In a few cases this has led to the introduction of a Foundation Year or Year Zero of integrated education at some institutions which extends the length of a degree of the same standard by one year.
2. In designing this CC proposal it was recognised that apart from student differences entry points to study programmes in engineering varied considerably between countries and classes of institution. However there was general agreement within the SEFI-MWG as to the demarcation criteria between mathematical topics that are believed to be at degree level and those which constitute prerequisite study. All such topics are itemised in the CC contents as a guide to institutions in quantifying the length of study programmes, recognising that the inclusion of prerequisite material within the degree must extend that programme.

4.3 METHODS OF TEACHING

1. Mathematics possesses a hierarchy and orthodoxy and the ability to concatenate a large amount of intellectual information per symbol string. Students will always be required in some measure to unpackage such information. For this reason it is expected that the traditional methods of human interaction between student and teacher, namely lecture, tutorial and example class will be required into the foreseeable future especially to support the major technological developments that are likely in the next ten to twenty years. It is also expected that such methods will be needed at every educational level in that human advice and direction can both provide motivation and specifically target student problems without resort to a time-wasting circuitous help search that machine intelligence might afford. However the 1990s seem very likely to establish mathematics as a laboratory subject in that part of the teacher's role will be to assist in the intelligent extraction of information from computer output in its various forms.
2. Computer and video enhancement of lectures and tutorials is always welcome if properly integrated in measured amounts and made directly relevant to the topic in question. Such enhancement has been built into teaching, often in set-piece format since the 1950s, but planning, logistics and cost are as formidable problems as ever. This makes such ventures highly desirable but very difficult without resources of all kinds, particularly staff time, and due account of this will always need to be taken.

4.4 THE TEACHER

1. Mathematics as taught to engineers needs to stand alone to maintain cohesion and to reinforce the notions of when and where to go to seek deeper understanding and expert advice. Engineers must within reasonable limitation be able to think in mathematical terms within the kernel of an engineering model. They should have competent motivated teachers, working in the field of mathematics and its applications, preferably with an associated research interest. This would ensure that the engineering mathematics taught would be open to expertise, innovation and development. If the teaching is based in a dedicated mathematics or statistics department, an environment should be created to encourage the proper use of human resources. Where engineering mathematics teaching is provided by user engineering departments, there may be a risk of omission of general principles, nugatory duplication of others, inexperienced and old-fashioned usage, and an emphasis on rapidly ageing training methods of approach at the expense of time-proven principles of education. These features are most likely to occur in the face of diminishing resources.

4.5 MODELLING AND ENGINEERING APPLICATIONS

1. There is a cornerstone requirement for engineers to model and to be able to solve modelled problems. Most writers agree that mathematical modelling is perhaps the most constricted psychological bottleneck in the entire mathematical learning

process and that the debate upon how to teach it is likely to continue and may never be resolved. Opinions among educators not surprisingly are deeply divided between those who favour a top-down approach followed by skill learning to those who feel that skills are paramount and that the teaching of modelling is both vague and wasteful. Such diverse opinions have been voiced on a number of occasions, but the SEFI-MWG believes that students should ideally possess a prior knowledge that fully equals or exceeds the mathematical requirements to solve a modelled problem. Failure to meet these requirements could frustrate and severely demotivate students and would thus be counter-productive. Having said this however it is most important to give a clear brief to those who are to teach modelling, irrespective of the educational stage.

2. By way of being a little more specific, an engineering or scientific application is useful at lower levels though trivial applications probably distract weaker students and are best avoided. At higher levels, eg solving systems of ordinary differential equations, a large amount of scope is open in engineering and should be exploited in the form of demonstrated models. The methods of approach to such problems will inevitably inculcate modelling skills into students. In statistics however meaningful applications are available at the lowest level and should be exploited.
3. Many forms of modelling are essential to engineer formation and an enhancement to the educative process could be the inclusion of a specific study of mathematical modelling techniques. Experience has shown that a successful way of teaching mathematical modelling is to have equal input from both mathematicians and engineers.

4.6 ALLOTTED TEACHING TIME

1. It is well recognised that mathematics teaching possesses no easily etched immediate pay-off and when taught to users, such as engineers, is always under pressure of time. This pattern seems to be true to varying degrees in all the participating SEFI countries and many of the institutions. The pressure seriously conditions what is taught and is most acutely felt in areas where time requirements are diffuse, eg laboratory work and modelling, and in later stages of the curriculum if the mathematical requirements on entry, ie entry points, are low. Course designers need to take special care, even making provision for load shedding and ranking topics into a priority, should they be forced to make reductions. The density of information presents a lower limit on all occasions and the hierarchical nature of the subject requires careful sequencing at times, making it difficult to quantify the penalties of postponement or cancellation of curriculum topics.
2. Noting that there is an in-built cost to education the pay-off often comes later in life with sound concepts revisited when an adaptation to new technology is required. As an example, many of the electrical engineers trained in Europe in the 1950s and 1960s found themselves mathematically fully prepared to cope with the knowledge which underlies digital technology.

3. Recommendations as to teaching time are given in the commentary upon the Core Curriculum.

4.7 LANGUAGE

Most engineers in the SEFI countries, at least at the lower levels, are taught in their own language, or in a major adjacent continental European language such as German or French. English features increasingly further up the degree, particularly in literature and almost exclusively in computer applications. The trend towards the increasing use of English seems likely to continue as technology expands and the recent decisions of the East European countries to supplant English for Russian as their first foreign language, notwithstanding the scientific German of pre-war times, may bring about the international adoption of English much more widely post 1992. Nonetheless the presentation of English, French and German versions of this document is aimed at providing maximum accessibility to professional engineers and their educators alike.

4.8 CONTINUING EDUCATION

1. Engineers, like other professionals, go through many programmes of formal or informal continuing education in order to keep abreast of developments in their field. This has been the practice in the past, and the signs are that even greater demands will be made of future engineering graduates in this respect. For example, advances in control theory and reliability engineering are presenting new intellectual challenges, and an informed approach to experimental design has become an important skill for engineers involved in the validation and testing of products or processes.
2. The CC is designed so that a common basic education in mathematics can be standardised and assured so that a genuine foundation can be laid, which will both enable the graduate to understand current technology and techniques, and facilitate the future process of continuing education. It is intended that an engineer whose study is based upon this CC can take career courses later in life, such as the European Advanced Short Course Programme which SEFI co-sponsors, which themselves can be designed with common mathematical standards in mind. Much time and expense is nowadays being incurred in running engineering career courses which make good shortfalls in mathematical education as well as undertaking the necessary revision of mathematical topics. Mature students find new mathematical learning particularly difficult and can be deterred from taking career courses which possess such a content. The hope is that the common European standards offered by the CC will in the longer term facilitate movement, motivate mature students into retraining without fear, increase access to career courses and possibly reduce the length of such courses. All this should make for much greater economic efficiency and effectiveness in the industrial community.

5 THE CORE CURRICULUM: PERSPECTIVE AND COMMENTARY

5.1 PERSPECTIVE

The SEFI-MWG was formed in 1982 to address the changing role of mathematics in engineering education. Seminars were held in each of the years 1984-8 to investigate development and innovation, the impact of computers, and to examine the mathematical toolkit of engineers. The proceedings are published in [2] to [7]. The use of computers, and particularly micro-computers, both in and out of the classroom was providing lecturers and students alike with a wide variety of services ranging from user-friendly conventional computation to computer algebra, interactive video and computer enhanced learning. The seminars concluded that the portfolio of engineering mathematics has widened so much in the twenty years since [1] was published, that the following areas of mathematics and their applications needed to be separately addressed:

- Analysis and Calculus
- Linear Algebra
- Discrete Mathematics
- Probability and Statistics

and that numerical methods should be integrated into the Curriculum.

Sub-Working Groups were set up to examine each of these four areas and documents were prepared amid much international liaison and dialogue, to be agreed at the SEFI-MWG Colloquium in Kassel, FRG in November 1989 at which nine European Countries were represented. The present CC proposal has emerged directly from these agreements and follow-up discussions, and is aimed at the engineer of the 1990s and his or her education for change in a world of emerging technology.

5.2 COMMENTARY

1. Of all the CC subjects Discrete Mathematics is the one which possesses the most distinct and exclusive hierarchy. Coupled with its enormous surge in importance in recent time and its central role in software engineering, it clearly deserves independent consideration.
2. A similar view can be taken of Linear Algebra, even though its associations with older classical mathematics are stronger, in that software packages to solve linear engineering and physically based problems have evolved in the forefront of all software development. This has led to a much wider use of sophisticated linear algebraic terminology across the entire spectrum of engineering literature.
3. The case for Probability and Statistics can also be argued partly on the grounds of increased software availability but this alone is insufficient to justify a separate Curriculum. The engineer does however need to understand the meaning of system or product reliability, simulation usage and the essential difference between

theoretical models whose solution is totally deterministic vis-vis practical problems or observational results in which random effects occur. Recognising too that decision making in engineering management rests all too often upon statistical data analysis and its off-shoots such as sampling and hypothesis testing it seems that a separate statistical education is needed.

4. Numerical methods are much more accessible at an earlier stage of study than they were only a very few years ago, and in view of their ultimate relevance to solving all complex engineering problems there is everything to be gained from their inclusion within the Curriculum. As such methods are now working tools many teachers believe that they should be fully integrated within the CC as well.
5. Discrete Mathematics, Linear Algebra and Probability and Statistics all have the advantage that they can be started from first principles by any engineering student at the outset of a degree. In each of these a hierarchical sequence needs to be followed throughout the Curriculum though there is some scope for diversity. The overall quantity of material, not exceeding 40 to 60 hours of study time in each of the three subjects, should not present major timetabling or interfacing problems with other subjects, mathematical or otherwise. Each of the three can be considered as a Core Curriculum element itself with an advanced Elective Course option for certain specialisations of engineer.
6. The Analysis and Calculus element is expected to cover about half of the Core Curriculum, ie about 120 to 140 hours of study time. Prior knowledge at the outset of the degree is essential and rests upon most of the student's entire knowledge of pure mathematics. It is therefore necessary to define minimum pre-requisite study in pure mathematics below degree level, or Core Level Zero. The Core Curriculum itself has a steep hierarchy which necessitates careful sequencing and integration with all other study. A questionnaire designed following the 5th SEFI- MWG Seminar, [8], was circulated to all the delegates, together with certain recipients in the USA, Australia and Eastern Europe, and this identified two hierarchical stages to the Core Curriculum in Analysis and Calculus, since designated Core Level One and Core Level Two. However these have been combined into a single CC to maintain consistency with the other three curricula and to allow institutions to decide their own hierarchical levels. It is also highlighted what should constitute the Elective Course topics though no list of elective topics can be exhaustive.
7. The recommended Core Curriculum of 220 to 320 hours of mathematics represents an ABSOLUTE MINIMUM of study aimed at encompassing the basic mathematical needs of engineering undergraduate students at all European institutions. This time includes only the common Core Curriculum 6.2 and does not allow for any elective course material or even pre-requisite study. Certain institutions, notably those with a longer study programme, or those with a strong mathematical engineering tradition will offer and will continue to offer a much more substantial Core. Such diversity can only enrich the educational opportunities which are built upon this minimum framework.

6 THE CORE CURRICULUM: CONTENT

6.1 ASSUMED PREREQUISITE KNOWLEDGE: CORE LEVEL ZERO

The proposed Core Curriculum will assume a previous education in mathematics with students having a sound understanding of arithmetic, classical algebra, geometry, trigonometry and elementary differential and integral calculus. Students without this previous knowledge should follow a foundation course prior to embarking on the core study programme. It does not assume any prior knowledge of Discrete Mathematics, Linear Algebra or Probability and Statistics. Specifically the prerequisite knowledge in mathematics assumed, includes the following:

1.1 Real Numbers and Algebra

- 1.1.1 Basic rules of arithmetic applied to the manipulation of real numbers.
- 1.1.2 Indices, logarithms, rational powers and real roots.
- 1.1.3 Manipulation of algebraic expressions including completion of the square and partial fractions.
- 1.1.4 Graphs of elementary functions including simple polynomial and rational functions. Roots of equations.
- 1.1.5 Solutions of systems of linear algebraic equations.
- 1.1.6 Graphical and algebraic treatment of inequalities.
- 1.1.7 Logarithmic and exponential functions and their graphs.
- 1.1.8 The binomial theorem with integer exponent.

1.2 Trigonometry

- 1.2.1 Trigonometric functions, angular measure, graphs and identities.
- 1.2.2 Application to right-angled triangles.
- 1.2.3 Sine and cosine rules.
- 1.2.4 Solution of simple trigonometric equations.

1.3 Geometry

- 1.3.1 Euclidean geometry of the triangle, simple polygons, the circle and elementary solids.
- 1.3.2 Geometry of the straight line.
- 1.3.3 The circle in Cartesian co-ordinates.
- 1.3.4 The conic sections, ie ellipse, parabola and hyperbola in canonical form, and their parametric representation.

1.4 Differential Calculus of One Variable

- 1.4.1 Concept of the derivative and its interpretation as a rate of change and slope of curve.
- 1.4.2 Derivatives of standard functions, polynomial, exponential, logarithmic and trigonometric.
- 1.4.3 Rules of differentiation - product, quotient and function of function.
- 1.4.4 Elementary treatment of maximum, minimum and points of inflexion.
- 1.5 Integral Calculus of One Variable
 - 1.5.1 The concept of integration as the limit of a sum.
 - 1.5.2 Integration as the reverse process of differentiation.
 - 1.5.3 The integrals of standard functions.

6.2 COMMON CORE CURRICULUM

This material is to be core requirement for all engineering undergraduate courses. Topics are itemised under the headings of Analysis and Calculus, Linear Algebra, Discrete Mathematics and Probability and Statistics. Certain topics can possibly be included under more than one heading and institutions may prefer an alternative definition to the one given here, e.g. the method of least squares could be included under 2.2.2 (Matrix Algebra) rather than 2.4.9 (Statistics Miscellaneous).

1. Core Curriculum in Analysis and Calculus

- 2.1.1 Sequences and Series: Mathematical induction, sums of finite series, simple power series, convergence, divergence and simple test criteria applied to infinite series.
- 2.1.2 Functions: Concept of function, mapping notations, inverse functions, graphical representation. Graphs of $f(x+a)$, $f(ax)$ etc. Exponential, trigonometric and hyperbolic functions including inverses. Roots of equations including simple numerical methods (eg bisection and fixed point iteration for $x = g(x)$ form).
- 2.1.3 Complex Numbers: Algebraic and geometric foundation, de Moivre's theorem, roots of equations, exponential form, relationships between trigonometric and hyperbolic functions. Introduction to functions of a complex variable, loci problems.
- 2.1.4 Vector Algebra: Vector and scalar quantities, vector notation and arithmetic, dot and cross products, equations of lines and planes.
- 2.1.5 Differential Calculus: Review of definition and rules of differentiation, concepts of continuity and smoothness, derivatives of inverse functions, higher derivatives, Newton's method, numerical solution of equations by iteration. Taylor and Maclaurin expansions, approximations and asymptotic behaviour of functions, elementary numerical interpolation.

- 2.1.6 Integral Calculus: Definition of the integral as the limit of a sum, numerical integration. Fundamental theorem of calculus, indefinite integration as the reverse of differentiation, standard techniques of integration. Engineering applications of integration. Improper integrals, estimation, convergence and divergence.
- 2.1.7 Ordinary Differential Equations (O.D.E.'s): Differential equations in engineering modelling, first order differential equations and the role of boundary/initial conditions, numerical solutions, analytic solutions in standard cases. Classification of O.D.E.'s, order, degree, linear, constant coefficient, time invariant, non-constant coefficient, superposition, non-linear equations. Solution of second order linear constant coefficient O.D.E.'s, forced and unforced systems, transient and steady state responses, frequency response and resonance, 2x2 matrix representation and extension of numerical techniques.
- 2.1.8 Calculus of Several Variables: Pictorial representation of functions of two variables, partial derivatives and their geometric interpretation, Taylor series, max, min, and saddle points, generalisation to functions of more than two variables. Differentials, total derivatives and change of variables. Linear least squares fit for a set of data points. Multiple integrals, change of order and change of variable. Vector calculus, vector operators, line and surface integrals, integral theorems.
- 2.1.9 Fourier Series: Periodic functions, even and odd functions. Fourier expansions with a description of convergence.

2.2 Core Curriculum in Linear Algebra

- 2.2.1 Linear Spaces and Transformations: Spaces, linear independence, bases. Subspaces. Scalar product, measure and Euclidean norm. Linear transformation, image space, null space. Matrix representation, change of basis. Orthogonal transformations.
- 2.2.2 Matrix Algebra and Linear Equations: Matrix representation of linear equations, solution of linear equations, by elimination methods. Matrix algebra including inversion and concept of rank. Determinants, properties, operations. Decomposition methods. Consistency, and uniqueness of solution. Ill-conditioned systems. Use of software packages.
- 2.2.3 The Eigenvalue Problem: Engineering origins of eigenvalue problems. Algebraic methods for determining eigenvalues and eigenvectors. Numerical approach to evaluation, power method. Reduction of a matrix to diagonal and Jordan form. Symmetric matrices and quadratic forms. Use of software packages.

2.3 Core Curriculum in Discrete Mathematics

- 2.3.1 Mathematical Logic: Propositions, connectives, **AND, OR, IMPLICATION, NEGATION**, and truth-tables. Rules of inference, quantifiers, elementary concepts of proof and program correctness. Boolean algebra, introductory definition and relation to logic.

- 2.3.2 Sets and Functions: Sets, subsets and an introduction to cardinality. Operations on sets, laws of set theory (relation to Boolean algebra, switching and logic circuits). Definition of a language (no grammar), production rules. Definition of a function, domain, co-domain and range. Partial functions. Recursive definition of functions, $n!$ as an example.
- 2.3.3 Number Systems: Peano axioms and the natural numbers, the integers. Binary and hexadecimal systems, introduction to modular arithmetic. Computer arithmetic, rational and real numbers. Introduction to the concept of computability. Induction.
- 2.3.4 Recursion: Difference equations and their solution.
- 2.3.5 Relations and Finite State Machines: Finite state machines and the computing process. Definition of a relation, equivalent states of a finite-state machine.
- 2.3.6 Algorithms: Concept of proof of an algorithm. Examples using logic and induction. Ideas of complexity sorting and searching algorithms as examples. Recursive algorithms.
- 2.4 The Core Curriculum in Probability and Statistics
- 2.4.1 Collection of Data: Randomness and uncertainty. Graphical presentation of data. Numerical description of data (mean, standard deviation, median, quartile etc) EDA, box plots, stem and leaf diagrams. Relative frequency and its statistical features.
- 2.4.2 Probability: Probability as a 'degree of belief'. Two particular methods for its evaluation: frequentist and combinatorial.
- 2.4.3 Combinatorics: Enumeration, permutations and combinations, number of subsets, multiplication principle.
- 2.4.4 Basic Laws of Probability: $P(A \text{ or } B)$, $P(A \text{ and } B)$, $P(A|B)$. Bayes Theorem. Conditional probability as an inferential tool in statistics. Prior and posterior probabilities.
- 2.4.5 One-Dimensional Random Variables (probability distributions): Empirical and theoretical distributions. Basic distributions (for example, binomial, Poisson, Hypergeometric, exponential, Normal, Weibull, Gamma), and their use in the probabilistic modelling of engineering situations.
- 2.4.6 Two-Dimensional Random Variables (probability distributions): Conditional distributions, independence, linear combinations of random variables, correlation and regression. Extension to n dimensions. Weak form of the law of large numbers and the Central Limit Theorem.
- 2.4.7 Simulation: Random numbers and random number generators, the simulation of probability distributions.
- 2.4.8 Statistical Inference, Classical or Bayesian treatment as preferred.
Classical treatment: Point estimation of parameters. Unbiased estimator, consistent estimator and the maximum likelihood method. Unbiased estimation of expectation and variance. Distribution-free methods, confidence

interval for the median. Estimating parameters of specific distributions (exponential, Normal, Poisson, etc) and the intensity of a Poisson process. Confidence intervals for the mean and variance, and for the parameters of the distributions above.

Bayesian treatment: The parameters of a distribution as a random variable. Prior and posterior distributions of the parameter. The posterior distribution as a tool for estimating parameters and for hypothesis testing. Estimating parameters of specific distributions (exponential, Normal, Poisson, etc) and the intensity of a Poisson process. Bayesian confidence intervals for the mean and variance and for the parameters of the distributions above.

- 2.4.9 Miscellaneous: Linear regression. Use of linear regression techniques. Use of probability paper. Basic ideas of statistical quality control, experimental design, queueing theory and reliability.

6.3 ADDITIONAL CORE MATERIAL: ELECTIVE COURSES

This material will be core requirement for particular engineering undergraduate courses. The topic list is indicative and NOT A COMPLETE LIST. Again there is considerable overlap between the main topic areas which is partly overcome by grouping analysis and calculus together with linear algebra. Some institutions may prefer topics under different headings, e.g. 3.2.1 Algebraic Structure.

3.1 Calculus and Linear Algebra

- 3.1.1 Linear Algebra: Further numerical methods. Linear algebraic equations including iterative methods. Eigenvalue problem (eg Jacobi's method, Householder's method, LR and QR methods).
- 3.1.2 Ordinary Differential Equations: Further numerical methods for solution. Predictor-corrector methods, Runge-Kutta methods. Boundary value problems.
- 3.1.3 Laplace Transforms: Definition of transforms of standard functions, inverses, shift theorems. Application to solution of ordinary differential equations. Impulse and step response. Response of linear systems in terms of convolution. Transfer functions, poles and zeros, stability. Frequency response.
- 3.1.4 Z-Transforms: Sampled functions, definitions of Z-transforms, basic properties and shift theorems. Solution of difference equations. Filter design.
- 3.1.5 Fourier Analysis: Uniform convergence of Fourier series, Gibb's phenomena. Parseval's Theorem and power spectra. Exponential form of Fourier series. Other orthogonal series expansions (eg Legendre, Bessel, Walsh). Fourier transforms, basic properties including symmetry. Discrete Fourier transforms. Use of FFT algorithm. Frequency response for discrete time systems, aliasing.
- 3.1.6 Calculus of Several Variables: Constrained and unconstrained optimisation. Vector calculus, analysis of curves and surfaces.

- 3.1.7 Complex Variable: Function of a complex variable. Cauchy-Riemann conditions, mappings. Series expansion and contour integration.
- 3.1.8 Partial Differential Equations: Partial differential equations in engineering modelling. Classification of second order equations and the role of initial and boundary conditions. Method of characteristics. D'Alembert solution. Separation of variables and Fourier series solution. Numerical solution using finite differences and finite elements.
- 3.1.9 Matrix Analysis: Functions of a square matrix, Cayley-Hamilton theorem. State space representation of linear systems. Solution of $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$ and $\mathbf{x}(k+1) = \mathbf{Ax}(k) + \mathbf{Bu}(k)$.
- 3.1.10 Functional Analysis: Metric spaces. Normed linear spaces and inner product. Hilbert space. L2 spaces and integrals. Orthogonality and Completeness.

3.2 Discrete Mathematics

- 3.2.1 Algebraic Structure: Sets with operations, semigroups and groups, homomorphisms and isomorphisms, quotient structures, modular arithmetic.
- 3.2.2 Finite Rings and Fields: Rings, polynomial rings and irreducible polynomials, field constructions, discrete Fourier Transform.
- 3.2.3 Combinatorics and Graph Theory: Counting techniques, the product rule, inclusion - exclusion, generating functions, Polya's enumeration. Graphs, directed and undirected, trees. Existence problems, pigeonhole principle, experimental design, existence problem for graphs. Optimisation problems, matching and covering, flows in network, decision trees, spanning trees.
- 3.2.4 Lattices and Boolean Algebras: Partial orders and lattices, Boolean algebra and Boolean functions, minimisation of Boolean functions.

3.3 Probability and Statistics

- 3.3.1 Stochastic Processes: Markov chains, transition matrices, stationarity. Chapman - Kolmogorov equation, absorption.
- 3.3.2 Statistical Quality Control: Shewhart charts for process control. Mean, range and other charts. Cusum procedures. Taguchi methods. Evolutionary operation procedures.
- 3.3.3 Reliability: Failure rates, hazard and reliability functions. Mean time between failures (MTBF). Weibull and Gumbel distributions. Estimation.
- 3.3.4 Experimental Design: One and two way designs. Fixed and random effects. $2n$ and $3n$ designs. Randomised blocks and Latin Squares.
- 3.3.5 Queueing Theory and Discrete Simulation: Single and multiple server queues, M/M/1, M/M/n, M/G/1, M/G/n. Properties of queueing systems. Application of simulation to complex queueing systems.
- 3.3.6 Filtering and Control: Kalman filters, linear filters, feedback loops, smoothing. Communication systems, noise filtration, white noise.

- 3.3.7 Markov Processes and Renewal Theory: Markov processes, stationarity, spectral density function. Correlogram. The birth-death process, simple renewal models, immigration-emigration, parameter estimation, repair times and breakdown.
- 3.3.8 Statistical Inference: Hypothesis testing, power and size, Type I and Type II errors. Likelihood ratio test, generalised likelihood ratio.
- 3.3.9 Multivariate Analysis: Cluster analysis, discriminant analysis, principal component analysis, factor analysis. Introduction to multivariate analysis of variance.

7 CONCLUSIONS AND BENEFITS

European institutions are strongly urged to adopt the Core Curriculum proposal in educating professional engineers for change in the 1990s. The proposal has been kept circumspect amid international consensus so that each institution and its host nation can exercise flexibility in its implementation. If this CC proves successful further curricula in other engineering subjects should follow. The SEFI-MWG believes that many benefits will ensue if the CC takes hold not least of which is wider European integration in the respects covered. Briefly expressed the benefits are believed to be as follows:

1. There is sound economic sense in having a common standard in engineering education.
2. Wider educational benefits ensue in having a common standard.
3. Professional development of mature engineers via Career Courses or self-study can be made more accessible, more attractive, and by standardising format, more effective and economically efficient.
4. Movements and transfer of engineers and their skills will be facilitated on both national and international bases.
5. International recognition of engineering qualifications will be facilitated.
6. The general image and integrity of the engineering professions will be enhanced.
7. The professional intellectualism of engineering will be better recognised.

8 APPENDIX

8.1 PRINCIPAL CONTRIBUTORS

The following contributors have taken an active part in the various activities within the SEFI-MWG leading to the preparation of this report.

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