MATHEMATICS FOR THE EUROPEAN ENGINEER

A CURRICULUM FOR THE TWENTY-FIRST CENTURY

A Report by the

SEFI MATHEMATICS WORKING GROUP

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Preface – By the President of SEFI

Engineering is based on mathematics, physics and technological solutions. Allthese disciplines are equally important. Computing technology is often included in these basic sciences; it is however mathematical solutions embedded in electronics. In engineering, mathematics is indispensable, because no technical discipline can afford to ignore it. In a sense, this has always been true, but it is becoming ever more so as the complexity of engineering tasks grows and abstract information processes start playing an essential role in technology. The way mathematics is included in different engineering educational institutions curricula varies, but there are some basic requirements to be fulfilled.

The European Society for Engineering Education, SEFI, has brought together many of the leading teachers of mathematics to engineers in a very active working group. This SEFI Mathematics Working Group has now produced a report: *Mathematics for the European Engineer - a Curriculum for the twenty-first Century*. This Core Curriculum that is the fruit of the hard work and commitment of many experts, reflects the considered opinion of leaders in the field of mathematical education for engineering. So, although this curriculum cannot, and should not, legally be enforced, every professor and every engineering institution is completely safe in following it.

A previous publication was produced a decade ago, but the rapid changes since that time, not least in enormous computing power, which now is available to every student, require a new version. The changes in this document in comparison with the previous one also reflect the changes in the way learning results are observed.

With this publication, SEFI, as the leading and widest European association for the promotion and improvement of higher engineering education, is proud to offer not only the members of its network but also any other engineering education institutions, the advice, the knowledge and the recognised expertise of its Mathematics Working Group.

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Chapter 1 Introduction

SEFI (The European Society for Engineering Education) is a non-governmental organisation founded in 1973. Its aims are:

- to promote information on engineering education
- to improve communications and exchanges between teachers, researchers and students
- to develop cooperation between engineering educational institutions
- to promote cooperation between industry and educational institutions
- to contribute to the development and to the improvement of engineering education in the economic, social and cultural framework of Europe.

Many of the activities of SEFI are conducted through its working groups. One of these is the Mathematics Working Group (SEFI MWG) which was formally established in 1982. Among its aims are:

- to provide a forum for the exchange of views and ideas amongst those interested in engineering mathematics
- to promote a fuller understanding of the role of mathematics in the engineering curriculum, and its relevance to industrial needs
- to foster co-operation in the development of courses and support material, in collaboration with industry
- to recognise and promote the role of mathematics in the Continuing Education of Engineers.

As one means of fulfilling these aims the Mathematics Working Group has organised a sequence of seminars which bring together colleagues from several countries to share ideas and best practice. The first of these seminars took place in Kassel in 1984. Subsequent seminars have been held in Copenhagen (1985), Turin (1986), Gothenburg (1987), Plymouth (1988), Dublin (1990), Budapest (1991), Abisko (1991), Eindhoven (1993), Prague (1994 and 1995), Espoo (1998) and Miskolc (2000). A special seminar on geometry in engineering education was held in Bratislava in 1997.

At the seminar in Plymouth there was an impetus towards focusing the activity of the Working Group on the production of a Core Curriculum in Mathematics for the European Engineer. Following a small group meeting in Kassel in late 1989, a draft document was prepared and this underwent extensive refinement.

In 1992 SEFI published the document, *A Core Curriculum in Mathematics for the European Engineer* [**1**].

The appearance of this first revision to the SEFI Core Curriculum is timely. So much has changed in the mathematical education area in the eight years since the original Core Curriculum was published that it is rapidly becoming out of date. It is appropriate that the revised version of the SEFI core curriculum in mathematics should be published at the start of the new millennium since this is a suitable time to introduce changes. It is now becoming more widely acceptable to phrase a curriculum in terms of learning outcomes rather than a list of topics to be covered. Accordingly, this revision presents the curriculum in this style.

The ever more rapid development of educational technology places the curriculum under a tension. The increase in computer power available to the student cannot be ignored: at the very least it demands a radical re-think on the way in which topics in the curriculum could be presented.

This curriculum is intended as a benchmark by which higher education institutions in Europe can judge the mathematics provision in their engineering undergraduate degree programmes. It does not aim to be prescriptive. Some institutions will already cover more mathematics in their programmes and it is to be hoped that they will continue to do so. However, other institutions will fall short of the recommended curriculum and, whilst we would hope that they will be encouraged to increase their mathematics provision, the first step would be to take the appropriate parts of the curriculum and fit them into the time allowed.

The symbiosis between mathematics and engineering is a long-standing one. Almost all branches of engineering rely on mathematics as a language of description and analysis. On the other hand, mathematics has benefited from a steady flow of engineering problems which require solving and this has sometimes led to the development of new areas of mathematics. This symbiosis has, however, had its tensions where the education of engineers is concerned.

The ever more rapid pace of technological development has created a situation in which many engineers will require frequent updating in areas of their specialisation. This may involve the mastery of new techniques and the understanding of new theoretical concepts. A fluency with mathematics is an essential weapon in the modern graduate engineer's armoury.

There is an increasing need for the mutual recognition of engineering qualifications across Europe. There is a rich variety in the length and design of degree programmes in different countries and the provision of mathematics in these programmes is not immune from this variation. However, in increasingly more countries there is concern over the deterioration in the mathematical ability of new entrants to engineering degree programmes.

At a basic level, mathematical topics of particular importance to engineers include:

- a fluency and confidence with number
- a fluency and confidence with algebra
- a knowledge of trigonometric functions
- an understanding of basic calculus and its application to 'real-world' situations
- a proficiency with the collection, management and interpretation of data.

Only a few years ago it was assumed that such topics were thoroughly covered in pre-university education**.** In the United Kingdom, the main university entry qualifications are A levels, with pass grades denoted by A to E. Work by Lawson [**2**] has shown that there has been a marked decline in proficiency in some of the topics listed above amongst students entering university. For example, in 1997 only 54% of students with mathematics A level grade C, tested at the start of their university studies, could correctly identify the graph of the cosine function. In 1991 all comparably qualified students could do this.

This problem is not restricted to the United Kingdom, but is true in many other countries; see for example Brandell [**3**], de Guzman et al [**4**], Pfeiffle and Nairz-Wirth [**5**] and Frisius [**6**]. In the Czech and Slovak republics regular conferences on the mathematical education of students at the technical universities are held. The declining standard of mathematical knowledge and skills of new entrants to engineering degree courses is discussed; Jevik and Koukal [**7**], Pavloviova [**8**]. There are indications that, following the transformation of the school system to a form comparative to that of Western and

Northern Europe, the results achieved were lower than expected, especially in mathematics. At the 1998 meeting of the SEFI Mathematics Working Group in Finland the decline in entry competencies of students was a common theme, expressed by delegates from across Europe and beyond.

On its own this decline in key engineering mathematics skills amongst students who obtained reasonable entry qualifications would be a significant concern. However, it has been compounded by the trend across Europe to increase the numbers entering higher education. As a result, some students who are less well qualified have started courses to which, previously, they would not have been admitted.

Some subjects, such as media studies and business studies, have been able to maintain or even increase their required entry qualifications. In many countries computer science courses have seen a large increase in demand. Meanwhile engineering has grown increasingly unpopular and, consequently, it has struggled to find sufficient numbers of recruits with the desired level of entry qualification. On the other hand, some very able and well-qualified students continue to wish to study engineering and this has led to cohorts with a wide spread of mathematical preparedness and ability.

Universities have not been unaware of these problems and a range of measure have been taken to address them. These measures include:

1. reducing syllabus content, replacing some of the harder material with more revision (or, for some students, vision) of lower level work;
2. developing additional units of study;
3. establishing mathematics support centres;
4. doing nothing.

Each of these measures has its own disadvantages. Reducing syllabus content may help the weaker students to pass; however, removing more advanced material disadvantages the more able students who become less well-prepared for the more advanced and more analytical parts of their engineering study.

The use of additional units of study to cover basic material means that there is no need to remove some more advanced material from the curriculum. However, this approach has a number of practical difficulties. Ideally the basic material should be covered before the 'standard' mathematics course is started, but there is no time for this. There is also the political question of who pays for this extra tuition: the students, the engineering department or the mathematics department?

Mathematics support centres are outside the formal taught course structure. They provide students experiencing difficulties with supplementary resources and additional tuition. However, those working in such centres (one author of this report manages such a centre in his own institution) know that this approach is not reaching the root of the problem. The less well-prepared students need a systematic programme of study in order to give them a coherent body of mathematical knowledge rather than one riddled with gaps. Some of these gaps are in fundamental areas. For example, many students are unable to perform the arithmetic of fractions, yet they are expected to be able to use partial fractions in order to determine integrals and inverse Laplace transforms.

The option of doing nothing allows standards to be maintained since syllabus content and assessment are not changed. However, the result is an ever increasing failure rate as more and more students are not able to cope with the material presented to them.

The students who need the most help are in a very difficult situation. Their mathematical weakness means that they need to spend a significant amount of time either in studying an extra module or in the mathematics support centre to receive help in remedying the deficiencies of their mathematical background. However, because of these weaknesses they find many of their other engineering subjects difficult and so need to spend extra time trying to keep up in these subjects. As a consequence they tend to view any extra mathematics that they undertake as a way of dealing with isolated problems and not as a means of constructing a coherent mathematical knowledge base.

In the United Kingdom the Engineering Council, which oversees the professional development of engineers, has attempted to address some of these concerns by proposing a radical change in the approach to the tertiary education of engineers by revising its policy document *Standards and Routes To Registration* [**9**]. The new policy is to have a clear division between two groups: Incorporated Engineers and Chartered Engineers. Incorporated Engineers will have mastery of basic mathematical techniques and will be proficient in applying these to the solution of engineering problems. They will also be familiar with a range of modern mathematical technologies which they will use to solve real problems. Chartered Engineers will have a wider exposure to mathematics. In addition to mastery of basic techniques, they will need deeper understanding of mathematical principles and also of more advanced mathematics relevant to their particular engineering discipline. Although this strict division into two groups appears radical, it can be thought of as little more than the formalisation of differences that exist in many countries between polytechnics and technical universities.

2.1 The role of the computer

In the last thirty years it is fair to say that a revolution has taken place in the extent of computer power which is widely available. The pocket electronic calculator has supplanted four-figure tables and slide rules, while batch processing on the mainframe has given way to on-line computing, which in turn has been superseded by the microcomputer and the PC. There are two connected but distinct issues related to this expansion in the availability of computing power which are of considerable importance in relation to the engineering mathematics curriculum. The first is that new approaches to teaching and learning are made possible. The second is that enormously sophisticated mathematical software is now commonly available which routinely allows analysis of problems of such size and complexity that only a few years ago they would have been regarded as research activities.

It might have been expected that the computing revolution would have been matched by a similar revolution in the content and style of teaching mathematics. In some isolated instances moves have been made and are being made, but not to the extent that may have been envisaged.

There are some signs that the involvement of computers in the teaching of undergraduate engineering mathematics is beginning to gain momentum. However, the experience of the last thirty years warns that care must be taken. When the pocket calculator arrived we were told that there would be two advantages: first, the students would be relieved of hours of tedious calculation, leaving them free to concentrate on concepts and understanding; second, it was more likely that the calculations would be performed correctly. The reality is somewhat different. The most trivial of calculations is often sub-contracted to the machine, the students have little or no feel for what they are actually doing or to what precision they should quote their answers. What they can do is to obtain obviously unrealistic results more quickly and to more significant figures. There has also been a tendency to replace analytical reasoning by trial-and-error methods.

With the advent of the microcomputer and the PC a second chance to change radically the nature of the teaching of mathematics presented itself. This time, the stumbling-block was the lack of quality software. Now, many hours of effort and much money are being invested to try to take advantage of the power of the computer in the learning of mathematics: the aim is to enhance the understanding of concepts and the acquisition of skills. This is an area which is still very much in its infancy. Enthusiasts can point to items of success. Sceptics can highlight areas of failure. It is perhaps fair to say that an emerging consensus is that the computer should not be seen as a means of eliminating the human teacher nor of reducing the amount of student effort required to master the subject. Rather the computer should be used as a means of allowing a clearer and more motivating way of addressing certain topics.

As computers have become more powerful so the problems which are being tackled in the real world are becoming ever more complex, demanding more sophisticated numerical techniques. Problems which were taxing research mathematicians a few decades ago are now within the realms of possibility for the undergraduate engineer. With an ever-increasing demand on an already crowded curriculum the time available for practical computing is squeezed hard. Commercially available software packages have become more user-friendly. Therein lurks a danger. Always anxious to reduce the contact hours allocated to mathematics, engineering colleagues point to the widespread availability of these packages. No need to spend time on teaching methods like Simpson’s rule and Trapezoidal rule, they argue, since they are not used in practice; just use the packages available as ‘black boxes’.

Clearly, the computer is here to stay and what is needed is a sensible balance between the use of software and the ‘pen and paper’ manipulation which is so essential to an understanding of fundamental concepts. Euler's method would never be used in practice to numerically solve a differential equation. However, in carrying out a few steps of this method on a simple problem a student will learn much about the nature of numerical solutions. Much of this would probably be overlooked if all the students did was learn the appropriate syntax to invoke a differential equation solver in a commercial software package.

The increasingly rapid changes in computer technology make it virtually impossible to anticipate the facilities which will be available to the average student even by the end of the next decade. Therefore in presenting the curriculum we have not tried to do so. The curriculum as presented offers many opportunities for teaching the material in innovative ways, including the use of software packages. The extent to which they are employed will depend *inter alia* on the facilities available at an institution. Furthermore, the curriculum also specifically includes learning objectives which require the students to learn how to use mathematical software to carry out tasks which are plainly most efficiently done by computer (such as solving large systems of simultaneous equations). It is important that students do not just learn the relevant commands in the software package available to them. They must learn to use these packages discerningly, from a base of mathematical knowledge that will inform them when the answers may be unreliable (such as in the case of near singularity). Students require a personal knowledge of mathematics to be able to use mathematical software reliably and effectively.

5. Overview

In this chapter the proposed Core Curriculum is presented. It has been arranged in a structure which has four levels. These levels represent an attempt to reflect the hierarchical structure of mathematics and the way in which mathematics can be linked to real applications of ever-greater sophistication as the student progresses through the engineering degree programme.

A schematic diagram of the proposed structure is shown in Figure 1. Note that there is a central core of material essential for all engineering graduates. Typically this core material would be covered by the end of the first year but the teaching of some of it might extend into the second year of a course. Realistic pre-requisite knowledge (Core Zero) would be assumed. Core Zero, as set out in Section 3.2, does not represent the minimum which can be assumed in every European country. Instead it is covers those topics which make up an essential foundation for Core Level 1 and beyond. It is likely that most institutions will need to teach explicitly some of Core Zero topics whilst other institutions may have a parallel programme of support classes or clinics to help students who may be deficient in some areas.

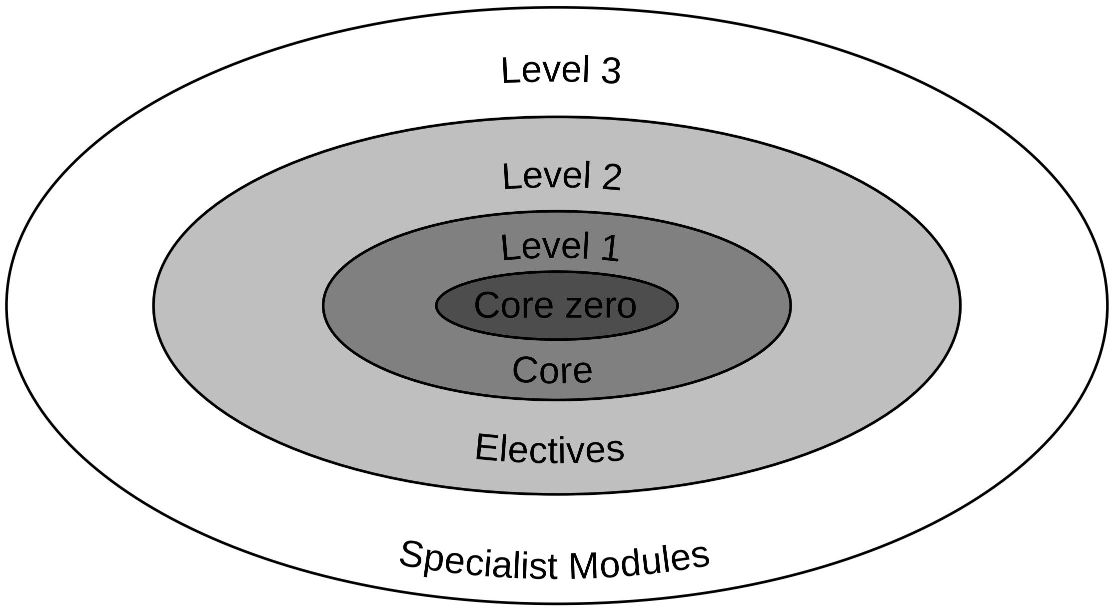

Figure 1 Schematic diagram of the proposed structure

Core Zero is specified in Section 3.2 in considerable detail. We make no apology for this: it comprises such essential material that no engineering student can afford to be deficient in these topics. The knowledge and skills in mathematics of a student entering tertiary education is not easily predicted from the qualifications achieved prior to entry and some kind of diagnostic testing and additional support may be needed. This support may well be needed throughout the first year of an engineering degree programme.

Core level 1 comprises the knowledge and skills which are necessary in order to underpin the general Engineering Science that is assumed to be essential for most engineering graduates. Items of basic knowledge will be linked together and simple illustrative examples will be used. It should be pointed out that the mathematical needs of Computer Science and Software Engineering are markedly different from all other branches of engineering. This core curriculum is only of limited use for such courses.

Level 2 comprises specialist or advanced knowledge and skills which are considered essential for individual engineering disciplines. Synoptic elements will link together items of knowledge and the use of simple illustrative examples from real-life engineering.

Level 3 comprises highly specialist knowledge and skills which are associated with advanced levels of study and incorporates synoptic mathematical theory and its integration with real-life engineering examples.

Students would progress from the core in mathematics by studying more subject-specific compulsory modules (electives). These would normally build upon the core modules and be expected to correspond to the outcomes associated with level 2 material. Such electives may build additionally on level 1, requiring knowledge of more advanced skills, and may link level 1 skills or introduce additional more engineering-specific related topics. An example of the first mode is that Mechanical Engineering students may need to study the vibration of mechanical systems through the applied use of systems of ordinary differential equations. Here, topics that build on foundations of differential calculus, complex numbers and matrix analysis might be expected to be covered in level 1, as these are topics probably learnt in isolation and without reference to specific engineering application. Alternatively an Electronic Engineering student may be required to learn elements of discrete mathematics directly relevant to the design and study of computer systems; these would probably be unsuitable for core material for all engineers. This is not to say that discrete mathematics should not be taught at level 1, but the context and outcomes need to be clearly discernible within the level. Typically level 2 modules would be distributed within the second or third year of an Engineering course due to the logistics of level 1 pre-requisites.

Students within the more numerate Engineering disciplines might be expected to take further more specialised modules incorporating mathematics on an optional basis, aimed to help match their career aspirations with appropriate theoretical formation. These modules will be at an advanced level, making use of appropriate technology, and heavily influenced with examples from engineering. Teaching of these level 3 modules would be most appropriate in year 3 or 4 of a degree course. It is likely that many of these topics already exist within specialist engineering courses and typically the mathematics is embedded and taught by engineers, mathematicians or both. For some programmes meeting the highest requirements, students might be expected to study some topics close to current areas of research where the available techniques and tools may well be mathematically based.

Within the three main levels the material has been arranged under five sub-headings: analysis and calculus, discrete mathematics, geometry, linear algebra, statistics and probability. There is no intention to prescribe how the topics in the Core Curriculum should be ordered: what is offered here is a convenient grouping of items. In many cases a topic could have been placed under one of the other sub-headings

The curriculum is specified in terms of content and learning objectives. This makes the document longer, but it makes more explicit exactly what is required and is therefore more transparent for both teacher and learner.

3.2 Core Zero

The material in this section is the material which ideally should have been studied before entry to an undergraduate engineering degree programme. However, it is recognised that whilst there is some commonality across Europe over what is studied in pre-university mathematics, there are also a number of areas of difference. Core Zero does not consist of just those topics which are taught in school in all European countries, rather it contains material which together forms a solid platform on which to build a study of engineering mathematics at university. A consequence of this is that in many countries it will be necessary to cover some Core Zero material during the first year of a university engineering course.

The material in Core Zero has been grouped into five areas: Algebra, Analysis & Calculus, Discrete Mathematics, Geometry & Trigonometry and Statistics & Probability. These relate to the five areas in each of the three main levels of the curriculum: Analysis & Calculus, Discrete Mathematics, Geometry, Linear Algebra and Statistics & Probability.

3.2.1 Algebra

Arithmetic of real numbers

As a result of learning this material you should be able to

- carry out the operations add, subtract, multiply and divide on both positive and negative numbers
- express an integer as a product of prime factors
- calculate the highest common factor and lowest common multiple of a set of integers
- obtain the modulus of a number
- understand the rules governing the existence of powers of a number
- combine powers of a number
- evaluate negative powers of a number
- express a fraction in its lowest form
- carry out arithmetic operations on fractions
- represent roots as fractional powers
- express a fraction in decimal form and vice-versa
- carry out arithmetic operations on numbers in decimal form
- round numerical values to a specified number of decimal places or significant figures
- understand the concept of ratio and solve problems requiring the use of ratios
- understand the scientific notation form of a number
- manipulate logarithms
- understand how to estimate errors in measurements and how to combine them.

Algebraic expressions and formulae

As a result of learning this material you should be able to

- add and subtract algebraic expressions and simplify the result
- multiply two algebraic expressions, removing brackets
- evaluate algebraic expressions using the rules of precedence
- change the subject of a formula
- distinguish between an identity and an equation
- obtain the solution of a linear equation
- recognise the kinds of solution for two simultaneous equations
- understand the terms direct proportion, inverse proportion and joint proportion
- solve simple problems involving proportion
- factorise a quadratic expression
- carry out the operations add, subtract, multiply and divide on algebraic fractions
- interpret simple inequalities in terms of intervals on the real line
- solve simple inequalities, both geometrically and algebraically
- interpret inequalities which involve the absolute value of a quantity.

Linear laws

As a result of learning this material you should be able to

- understand the Cartesian co-ordinate system
- plot points on a graph using Cartesian co-ordinates
- understand the terms ‘gradient’ and ‘intercept’ with reference to straight lines
- obtain and use the equation $y = mx + c$
- obtain and use the equation of a line with known gradient through a given point
- obtain and use the equation of a line through two given points
- use the intercept form of the equation of a straight line
- use the general equation $ax + by + c = 0$
- determine algebraically whether two points lie on the same side of a straight line
- recognise when two lines are parallel
- recognise when two lines are perpendicular
- obtain the solution of two simultaneous equations in two unknowns using graphical and algebraic methods
- interpret simultaneous linear inequalities in terms of regions in the plane
- reduce a relationship to linear form.

Quadratics, cubics, polynomials

As a result of learning this material you should be able to

- recognise the graphs of $y = x^2$ and $y = -x^2$
- understand the effects of translation and scaling on the graph of $y = x^2$
- rewrite a quadratic expression by completing the square
- use the rewritten form to sketch the graph of the general expression $ax^2 + bx + c$
- determine the intercepts on the axes of the graph of $y = ax^2 + bx + c$
- determine the highest or lowest point on the graph of $y = ax^2 + bx + c$
- sketch the graph of a quadratic expression
- state the criterion that determines the number of roots of a quadratic equation
- solve the equation $ax^2 + bx + c = 0$ via factorisation, by completing the square and by the formula
- recognise the graphs of $y = x^3$ and $y = -x^3$
- recognise the main features of the graph of $y = ax^3 + bx^2 + cx + d$
- recognise the main features of the graphs of quartic polynomials
- state and use the remainder theorem
- derive the factor theorem
- factorise simple polynomials as a product of linear and quadratic factors.

3.2.2 Analysis and Calculus

Functions and their inverses

As a result of learning this material you should be able to

- define a function, its domain and its range
- use the notation $f(x)$
- determine the domain and range of simple functions
- relate a pictorial representation of a function to its graph and to its algebraic definition
- determine whether a function is injective, surjective, bijective
- understand how a graphical translation can alter a functional description
- understand how a reflection in either axis can alter a functional description
- understand how a scaling transformation can alter a functional description
- determine the domain and range of simple composite functions
- use appropriate software to plot the graph of a function
- obtain the inverse of a function by a pictorial representation, graphically or algebraically
- determine the domain and range of the inverse of a function
- determine any restrictions on $f(x)$ for the inverse to be a function
- obtain the inverse of a composite function
- recognise the properties of the function $1/x$
- understand the concept of the limit of a function.

Sequences, series, binomial expansions

As a result of learning this material you should be able to

- define a sequence and a series and distinguish between them
- recognise an arithmetic progression and its component parts
- find the general term of an arithmetic progression
- find the sum of an arithmetic series
- recognise a geometric progression and its component parts
- find the general term of a geometric progression
- find the sum of a finite geometric series
- interpret the term ‘sum’ in relation to an infinite geometric series
- find the sum of an infinite geometric series when it exists
- find the arithmetic mean of two numbers
- find the geometric mean of two numbers
- obtain the binomial expansions of $(a+b)^s$, $(1+x)^s$ for s a rational number
- use the binomial expansion to obtain approximations to simple rational functions.

Logarithmic and exponential functions

As a result of learning this material you should be able to

- recognise the graphs of the power law function
- define the exponential function and sketch its graph
- define the logarithmic function as the inverse of the exponential function
- use the laws of logarithms to simplify expressions
- solve equations involving exponential and logarithmic functions
- solve problems using growth and decay models.

Rates of change and differentiation

As a result of learning this material you should be able to

- define average and instantaneous rates of change of a function
- understand how the derivative of a function at a point is defined
- recognise the derivative of a function as the instantaneous rate of change
- interpret the derivative as the gradient at a point on a graph
- distinguish between ‘derivative’ and ‘derived function’
- use the notations $\frac{dy}{dx}$, $f'(x)$, y' etc.
- use a table of the derived functions of simple functions
- recall the derived function of each of the standard functions
- use the multiple, sum, product and quotient rules
- use the chain rule
- relate the derivative of a function to the gradient of a tangent to its graph
- obtain the equation of the tangent and normal to the graph of a function.

Stationary points, maximum and minimum values

As a result of learning this material you should be able to

- use the derived function to find where a function is increasing or decreasing
- define a stationary point of a function
- distinguish between a turning point and a stationary point
- locate a turning point using the first derivative of a function
- classify turning points using first derivatives
- obtain the second derived function of simple functions
- classify stationary points using second derivatives.

Indefinite integration

As a result of learning this material you should be able to

- reverse the process of differentiation to obtain an indefinite integral for simple functions
- understand the role of the arbitrary constant
- use a table of indefinite integrals of simple functions
- understand and use the notation for indefinite integrals

- use the constant multiple rule and the sum rule
- use indefinite integration to solve practical problems such as obtaining velocity from a formula for acceleration or displacement from a formula for velocity.

Definite integration, applications to areas and volumes

As a result of learning this material you should be able to

- understand the idea of a definite integral as the limit of a sum
- realise the importance of the Fundamental Theorem of the Calculus
- obtain definite integrals of simple functions
- use the main properties of definite integrals
- calculate the area under a graph and recognise the meaning of a negative value
- calculate the area between two curves
- calculate the volume of a solid of revolution
- use trapezium and Simpson's rules to approximate the value of a definite integral.

Complex numbers

As a result of learning this material you should be able to

- define a complex number and identify its component parts
- represent a complex number on an Argand diagram
- carry out the operations of addition and subtraction
- write down the conjugate of a complex number and represent it graphically
- identify the modulus and argument of a complex number
- carry out the operations of multiplication and division in both Cartesian and polar form
- solve equations of the form $z^n = a$, where a is a real number.

Proof

As a result of learning this material you should be able to

- distinguish between an axiom and a theorem
- understand how a theorem is deduced from a set of axioms
- appreciate how a corollary is developed from a theorem
- follow a proof of Pythagoras' theorem
- follow proofs of theorems for example, the concurrency of lines related to triangles and/or the equality of angles related to circles.

3.2.3 Discrete Mathematics

Sets

As a result of learning this material you should be able to

- understand the concepts of a set, a subset and the empty set
- determine whether an item belongs to a given set or not
- use and interpret Venn diagrams
- find the union and intersection of two given sets
- apply the laws of set algebra.

3.2.4 Geometry and Trigonometry

Geometry

As a result of learning this material you should be able to

- recognise the different types of angle
- identify the equal angles produced by a transversal cutting parallel lines
- identify the different types of triangle
- state and use the formula for the sum of the interior angles of a polygon
- calculate the area of a triangle
- use the rules for identifying congruent triangles
- know when two triangles are similar
- state and use Pythagoras' theorem
- understand radian measure
- convert from degrees to radians and vice-versa
- state and use the formulae for the circumference of a circle and the area of a disc
- calculate the length of a circular arc
- calculate the areas of a sector and of a segment of a circle
- quote formulae for the area of simple plane figures
- quote formulae for the volume of elementary solids: a cylinder, a pyramid, a tetrahedron, a cone and a sphere
- quote formulae for the surface area of elementary solids: a cylinder, a cone and a sphere
- sketch simple orthographic views of elementary solids
- understand the basic concept of a geometric transformation in the plane
- recognise examples of a metric transformation (isometry) and affine transformation (similitude)
- obtain the image of a plane figure in a defined geometric transformation: a translation in a given direction, a rotation about a given centre, a symmetry with respect to the centre or to the axis, scaling to a centre by a given ratio.

Trigonometry

As a result of learning this material you should be able to

- define the sine, cosine and tangent of an acute angle
- define the reciprocal ratios cosecant, secant and cotangent
- state and use the fundamental identities arising from Pythagoras' theorem
- relate the trigonometric ratios of an angle to those of its complement
- relate the trigonometric ratios of an angle to those of its supplement
- state in which quadrants each trigonometric ratio is positive (the CAST rule)
- state and apply the sine rule
- state and apply the cosine rule
- calculate the area of a triangle from the lengths of two sides and the included angle
- solve a triangle given sufficient information about its sides and angles
- recognise when there is no triangle possible and when two triangles can be found.

Co-ordinate geometry

As a result of learning this material you should be able to
- calculate the distance between two points
- find the position of a point which divides a line segment in a given ratio
- find the angle between two straight lines
- calculate the distance of a given point from a given line
- calculate the area of a triangle knowing the co-ordinates of its vertices
- give simple examples of a locus
- recognise and interpret the equation of a circle in standard form and state its radius and centre
- convert the general equation of a circle to standard form
- recognise the parametric equations of a circle
- derive the main properties of a circle, including the equation of the tangent at a point
- define a parabola as a locus
- recognise and interpret the equation of a parabola in standard form and state its vertex, focus, axis, parameter and directrix
- recognise the parametric equation of a parabola
- derive the main properties of a parabola, including the equation of the tangent at a point
- understand the concept of parametric representation of a curve
- use polar co-ordinates and convert to and from Cartesian co-ordinates.

Trigonometric functions and applications

As a result of learning this material you should be able to
- define the term periodic function
- sketch the graphs of $\sin x$, $\cos x$ and $\tan x$ and describe their main features
- deduce the graphs of the reciprocal functions cosec, sec and cot
- deduce the nature of the graphs of $a\sin x$, $a\cos x$, $a\tan x$
- deduce the nature of the graphs of $\sin ax$, $\cos ax$, $\tan ax$
- deduce the nature of the graphs of $\sin(x+a)$, $a+\sin x$, etc
- solve the equations $\sin x = c$, $\cos x = c$, $\tan x = c$
- use the expression $a\sin(\omega t+\phi)$ to represent an oscillation and relate the parameters to the motion
- rewrite the expression $a\cos\omega t + b\sin\omega t$ as a single cosine or sine formula.

Trigonometric identities

As a result of learning this material you should be able to
- obtain and use the compound angle formulae and double angle formulae
- obtain and use the product formulae
- solve simple problems using these identities.

3.2.5 Statistics and Probability

Data Handling

As a result of learning this material you should be able to
- interpret data presented in the form of line diagrams, bar charts, pie charts
- interpret data presented in the form of stem and leaf diagrams, box plots, histograms
- construct line diagrams, bar charts, pie charts, stem and leaf diagrams, box plots, histograms for suitable data sets
- calculate the mode, median and mean for a set of data items.

Probability

As a result of learning this material you should be able to
- define the terms ‘outcome’, ‘event’ and ‘probability’.
- calculate the probability of an event by counting outcomes
- calculate the probability of the complement of an event
- calculate the probability of the union of two mutually-exclusive events
- calculate the probability of the union of two events
- calculate the probability of the intersection of two independent events.

3.3 Core level 1

The material at this level builds on Core Zero and is regarded as basic to all engineering disciplines in that it provides the fundamental understanding of many mathematical principles. However, it is recognised that the emphasis given to certain topics within Core level 1 may differ according to the engineering discipline. So, for example, electrical and electronic engineers may cover some of the topics in Discrete Mathematics in greater depth than, say, Mechanical Engineers.

The material in Core level 1 can be used by engineers in the understanding and the development of theory and in the sensible selection of tools for analysis of engineering problems. This material will be taught in the early stages of a university programme. Noting the comment made in Section 3.2 it is possible that some of this material will be taught alongside or immediately after coverage of missing topics from Core Zero.

3.3.1 Analysis and Calculus

The material in this section covers the basic development of analysis and calculus consequent on the material in Core Zero.

Hyperbolic functions

As a result of learning this material you should be able to
- define and sketch the functions sinh, cosh, tanh
- sketch the reciprocal functions cosech, sech and coth
- state the domain and range of the inverse hyperbolic functions
- recognise and use basic hyperbolic identities
- apply the functions to a practical problem (for example, a suspended cable)
- understand how the functions are used in simplifying certain standard integrals.

Rational functions

As a result of learning this material you should be able to
- sketch the graph of a rational function where the numerator is a linear expression and the denominator is either a linear expression or the product of two linear expressions
- obtain the partial fractions of a rational function, including cases where the denominator has a repeated linear factor or an irreducible quadratic factor.

Complex numbers

As a result of learning this material you should be able to
- state and use Euler's formula
- state and understand De Moivre's theorem for a rational index
- find the roots of a complex number
- link trigonometric and hyperbolic functions
- describe regions in the plane by restricting the modulus and / or the argument of a complex number.

Functions

As a result of learning this material you should be able to
- define and recognise an odd function and an even function
- understand the properties 'concave' and 'convex'
- identify, from its graph where a function is concave and where it is convex
- define and locate points of inflection on the graph of a function
- evaluate a function of two or more variable at a given point
- relate the main features, including stationary points, of a function of 2 variables to its 3D plot and to a contour map
- obtain the first partial derivatives of simple functions of several variables
- use appropriate software to produce 3D plots and/or contour maps.

Differentiation

As a result of learning this material you should be able to
- understand the concepts of continuity and smoothness
- differentiate inverse functions
- differentiate functions defined implicitly
- differentiate functions defined parametrically
- locate any points of inflection of a function
- find greatest and least values of physical quantities.

Sequences and series

As a result of learning this material you should be able to
- understand convergence and divergence of a sequence
- know what is meant by a partial sum
- understand the concept of a power series
- apply simple tests for convergence of a series
- find the tangent and quadratic approximations to a function
- understand the idea of radius of convergence of a power series
- recognise Maclaurin series for standard functions
- understand how Maclaurin series generalise to Taylor series
- use Taylor series to obtain approximate percentage changes in a function.

Methods of integration

As a result of learning this material you should be able to
- obtain definite and indefinite integrals of rational functions in partial fraction form
- apply the method of integration by parts to indefinite and definite integrals
- use the method of substitution on indefinite and definite integrals
- solve practical problems which require the evaluation of an integral
- recognise simple examples of improper integrals
- use the formula for the maximum error in a trapezoidal rule estimate
- use the formula for the maximum error in a Simpson's rule estimate.

Applications of integration

As a result of learning this material you should be able to
- find the length of part of a plane curve
- find the curved surface area of a solid of revolution
- obtain the mean value and root-mean-square (RMS) value of a function in a closed interval
- find the first and second moments of a plane area about an axis
- find the centroid of a plane area and of a solid of revolution.

Solution of non-linear equations

As a result of learning this material you should be able to

- use intersecting graphs to help locate approximately the roots of non-linear equations
- use Descartes' rules of signs for polynomial equations
- understand the distinction between point estimation and interval reduction methods
- use a point estimation method and an interval reduction method to solve a practical problem
- understand the various convergence criteria
- use appropriate software to solve non-linear equations.

3.3.2 Discrete Mathematics

The material in this section covers the basic development of discrete mathematics consequent on the material in Core Zero.

Mathematical logic

As a result of learning this material you should be able to

- recognise a proposition
- negate a proposition
- form a compound proposition using the connectives AND, OR, IMPLICATION
- construct a truth table for a compound proposition
- construct a truth table for an implication
- verify the equivalence of two statements using a truth table
- identify a contradiction and a tautology
- construct the converse of a statement
- obtain the contrapositive form of an implication
- understand the universal quantifier “for all”
- understand the existential quantifier “there exists”
- negate propositions with quantifiers
- follow simple examples of direct and indirect proof
- follow a simple example of a proof by contradiction.

Sets

As a result of learning this material you should be able to

- understand the notion of an ordered pair
- find the Cartesian product of two sets
- define a characteristic function of a subset of a given universe
- compare the algebra of switching circuits to that of set algebra
- analyse simple logic circuits comprising AND, OR, NAND, NOR and EXCLUSIVE OR gates
- understand the concept of a countable set.

Mathematical induction and recursion

As a result of learning this material you should be able to

- understand (weak) mathematical induction
- follow a simple proof which uses mathematical induction
- define a set by induction
- use structural induction to prove some simple properties of a set which is given by induction.
- understand the concept of recursion
- define the factorial of a positive integer by recursion (any other suitable example will serve just as well).

Graphs

As a result of learning this material you should be able to

- recognise a graph (directed and/or undirected) in a real situation
- understand the notions of a path and a cycle
- understand the notion of a tree and a binary tree
- understand the notion of a binary tree.

3.3.3 Geometry

The material in this section covers the basic development of geometry consequent on the material in Core Zero.

Conic sections

As a result of learning this material you should be able to

- recognise the equation of an ellipse in standard form and state its foci, semiaxes and directrices
- recognise the parametric equations of an ellipse
- derive the main properties of an ellipse, including the equation of the tangent at a point
- recognise the equation of a hyperbola in standard form and find its foci, semiaxes and asymptotes
- recognise parametric equations of a hyperbola
- derive the main properties of a hyperbola, including the equation of the tangent at a point
- recognise the equation of a conic section in the general form and classify the type of conic section

3D co-ordinate geometry

As a result of learning this material you should be able to

- recognise and use the standard equation of a straight line in 3D
- recognise and use the standard equation of a plane
- find the angle between two straight lines
- find where two straight lines intersect
- find the angle between two planes
- find the intersection line of two planes
- find the intersection of a line and a plane
- find the angle between a line and a plane
- calculate the distance between two points, a point and a line, a point and a plane
- calculate the distance between two lines, a line and a plane, two planes
- recognise and use the standard equation of a singular quadratic surface (cylindrical, conical)
- recognise and use the standard equation of a regular quadratic surface (ellipsoid, paraboloid, hyperboloid).).

3.3.4 Linear Algebra

The material in this section covers the basic development of linear algebra consequent on the material in Core Zero.

Vector arithmetic

As a result of learning this material you should be able to

- distinguish between vector and scalar quantities
- understand and use vector notation
- represent a vector pictorially
- carry out scalar multiplication of a vector and represent it pictorially
- determine the unit vector in a specified direction
- represent a vector in component form (two and three components only).

Vector algebra and applications

As a result of learning this material you should be able to

- solve simple problems in geometry using vectors
- solve simple problems using the component form (for example, in mechanics)
- define the scalar product of two vectors and use it in simple applications
- understand the geometric interpretation of the scalar product
- define the vector product of two vectors and use it in simple applications
- understand the geometric interpretation of the vector product
- define the scalar triple product of three vectors and use it in simple applications
- understand the geometric interpretation of the scalar triple product.

Matrices and determinants

As a result of learning this material you should be able to

- understand what is meant by a matrix
- recall the basic terms associated with matrices (for example, diagonal, trace, square, triangular, identity)
- obtain the transpose of a matrix
- determine any scalar multiple of a matrix
- recognise when two matrices can be added and find, where possible, their sum
- recognise when two matrices can be multiplied and find, where possible, their product
- calculate the determinant of 2 x 2 and 3 x 3 matrices
- understand the geometric interpretation of 2 x 2 and 3 x 3 determinants
- use the elementary properties of determinants in their evaluation
- state the criterion for a square matrix to have an inverse
- write down the inverse of a 2 x 2 matrix when it exists
- determine the inverse of a matrix, when it exists, using row operations
- calculate the rank of a matrix
- use appropriate software to determine inverse matrices.

Solution of simultaneous linear equations

As a result of learning this material you should be able to

- represent a system of linear equations in matrix form
- understand how the general solution of an inhomogeneous linear system of *m* equations in *n* unknowns is obtained from the solution of the homogeneous system and a particular solution
- recognise the different possibilities for the solution of a system of linear equations
- give a geometrical interpretation of the solution of a system of linear equations
- understand how and why the rank of the coefficient matrix and the augmented matrix of a linear system can be used to analyse its solution
- use the inverse matrix to find the solution of 3 simultaneous linear equations
- understand the term ‘ill-conditioned’
- apply the Gauss elimination method and recognise when it fails
- understand the Gauss-Jordan variation
- use appropriate software to solve simultaneous linear equations.

Least squares curve fitting

As a result of learning this material you should be able to

- define the least squares criterion for fitting a straight line to a set of data points
- understand how and why the criterion is satisfied by the solution of $A^TAx = A^Tb$
- understand the effect of outliers
- modify the method to deal with polynomial models
- use appropriate software to fit a straight line to a set of data points.

Linear spaces and transformations

As a result of learning this material you should be able to

- define a linear space
- define and recognise linear independence
- define and obtain a basis for a linear space
- define a subspace of a linear space and find a basis for it
- define scalar product in a linear space
- understand the concept of measure
- define the Euclidean norm
- define a linear transformation between two spaces; define the image space and the null space for the transformation
- derive the matrix representation of a linear transformations
- understand how to carry out a change of basis
- define an orthogonal transformation
- apply the above matrices of linear transformations in the Euclidean plane and Euclidean space
- recognise matrices of Euclidean and affine transformations: identity, translation, symmetry, rotation and scaling.

3.3.5 Statistics and Probability

The material in this section covers the basic development of statistics and probability consequent on the material in Core Zero.

Data Handling

As a result of learning this material you should be able to

- calculate the range, inter-quartile range, variance and standard deviation for a set of data items
- distinguish between a population and a sample
- know the difference between the characteristic values (moments) of a population and a sample
- construct a suitable frequency distribution from a data set
- calculate relative frequencies
- calculate measures of average and dispersion for a grouped set of data
- understand the effect of grouping on these measures.

Combinatorics

As a result of learning this material you should be able to

- evaluate the number of ways of arranging unlike objects in a line
- evaluate the number of ways of arranging objects in a line, where some are alike
- evaluate the number of ways of arranging unlike objects in a ring
- evaluate the number of ways of permuting r objects from n unlike objects
- evaluate the number of combinations of r objects from n unlike objects
- use the multiplication principle for combinations.

Simple probability

As a result of learning this material you should be able to

- interpret probability as a degree of belief
- understand the distinction between *a priori* and *a posteriori* probabilities
- use a tree diagram to calculate probabilities
- know what conditional probability is and be able to use it (Bayes' theorem)
- calculate probabilities for series and parallel connections.

Probability models

As a result of learning this material you should be able to

- define a random variable and a discrete probability distribution
- state the criteria for a binomial model and define its parameters
- calculate probabilities for a binomial model
- state the criteria for a Poisson model and define its parameters
- calculate probabilities for a Poisson model

- state the expected value and variance for each of these models
- understand when a random variable is continuous
- explain the way in which probability calculations are carried out in the continuous case.

Normal distribution

As a result of learning this material you should be able to

- handle probability statements involving continuous random variables
- convert a problem involving a normal variable to the area under part of its density curve
- relate the general normal distribution to the standardised normal distribution
- use tables for the standardised normal variable
- solve problems involving a normal variable using tables.

Sampling

As a result of learning this material you should be able to

- define a random sample
- know what a sampling distribution is
- understand the term ‘mean squared error’ of an estimate
- understand the term ‘unbiasedness’ of an estimate

Statistical inference

As a result of learning this material you should be able to

- apply confidence intervals to sample estimates
- follow the main steps in a test of hypothesis.
- understand the difference between a test of hypothesis and a significance test (p-value)
- define the level of a test (error of the first kind)
- define the power of a test (error of the second kind)
- state the link between the distribution of a normal variable and that of means of samples
- place confidence intervals around the sample estimate of a population mean
- test claims about the population mean using results from sampling
- recognise whether an alternative hypothesis leads to a one-tail or a two-tail test
- compare the approaches of using confidence intervals and hypothesis tests.

3.4 Level 2

The material at this level builds on Core Level 1. The material is now advanced enough for simple real engineering problems to be addressed. The material in this level can no longer be regarded as essential for every engineer (hence the omission of 'Core' from the title of this level). Different disciplines will select different topics from the material outlined here. Furthermore, different disciplines may well select different amounts of material from Level 2. Those engineering disciplines which are more mathematically-based, such as electrical and chemical engineering, will require their students to study more Level 2 topics than other disciplines, such as manufacturing and production engineering which are less mathematically-based.

3.4.1 Analysis and Calculus

The material in this section covers the basic development of analysis and calculus consequent on the material in Core Level 1.

Ordinary differential equations

As a result of learning this material you should be able to

- understand how rates of change can be modelled using first and second derivatives
- recognise the kinds of boundary condition which apply in particular situations
- distinguish between boundary and initial conditions
- distinguish between general solution and particular solution
- understand how existence and uniqueness relate to a solution
- classify differential equations and recognise the nature of their general solution
- understand how substitution methods can be used to simplify ordinary differential equations
- use an appropriate software package to solve ordinary differential equations.

First order ordinary differential equations

As a result of learning this material you should be able to

- recognise when an equation can be solved by separating its variables
- obtain general solutions of equations by applying the method
- obtain particular solutions by applying initial conditions
- recognise the common equations of the main areas of application
- interpret the solution and its constituent parts in terms of the physical problem
- understand the term ‘exact equation’
- obtain the general solution to an exact equation
- solve linear differential equations using integrating factors
- find and interpret solutions to equations describing standard physical situations
- use a simple numerical method for estimating points on the solution curve.

Second order equations - complementary function and particular integral

As a result of learning this material you should be able to

- distinguish between free and forced oscillation
- recognise linear second-order equations with constant coefficients and how they arise in the modelling of oscillation
- obtain the types of complementary function and interpret them in terms of the model
- find the particular integral for simple forcing functions
- obtain the general solution to the equation
- apply initial conditions to obtain a particular solution

- identify the transient and steady-state response
- apply boundary conditions to obtain a particular solution, where one exists
- recognise and understand the meaning of ‘beats’
- recognise and understand the meaning of resonance.

Functions of several variables

As a result of learning this material you should be able to
- define a stationary point of a function of several variables
- define local maximum, local minimum and saddle point for a function of two variables
- locate the stationary points of a function of several variables
- obtain higher partial derivatives of simple functions of two or more variables
- understand the criteria for classifying a stationary point of a function of two variables
- obtain total rates of change of functions of two variables
- approximate small errors in a function using partial derivatives.

Fourier series

As a result of learning this material you should be able to
- understand the effects of superimposing sinusoidal waves of different frequencies
- recognise that a Fourier series approximation can be derived by a least squares approach
- understand the idea of orthogonal functions
- use the formulae to find Fourier coefficients in simple cases
- appreciate the effect of including more terms in the approximation
- interpret the resulting series, particularly the constant term
- comment on the usefulness of the series obtained.
- state the simplifications involved in approximating odd or even functions
- sketch odd and even periodic extensions to a function defined on a restricted interval
- obtain Fourier series for these extensions
- compare the two series for relative effectiveness
- obtain a Fourier series for a function of general period.

Double integrals

As a result of learning this material you should be able to
- interpret the components of a double integral
- sketch the area over which a double integral is defined
- evaluate a double integral by repeated integration
- reverse the order of a double integral
- convert a double integral to polar co-ordinates and evaluate it
- find volumes using double integrals.

Further multiple integrals

As a result of learning this material you should be able to
- express problems in terms of double integrals
- interpret the components of a triple integral
- sketch the region over which a triple integral is defined
- evaluate a simple triple integral by repeated integration
- formulate and evaluate a triple integral expressed in cylindrical polar co-ordinates
- formulate and evaluate a triple integral expressed in spherical polar co-ordinates
- use multiple integrals in the solution of engineering problems.

Vector calculus

As a result of learning this material you should be able to
- obtain the gradient of a scalar point function
- obtain the directional derivative of a scalar point function and its maximum rate of change at a point
- understand the concept of a vector field
- obtain the divergence of a vector field
- obtain the curl of a vector field
- apply simple properties of the operator ∇
- know that the curl of the gradient of a scalar is the zero vector
- know that the divergence of the curl of a vector is zero
- define and use the Laplacian operator ∇^2.

Line and surface integrals, integral theorems

As a result of learning this material you should be able to
- evaluate line integrals along simple paths
- apply line integrals to calculate work done
- apply Green's theorem in the plane to simple examples
- evaluate surface integrals over simple surfaces
- use the Jacobian to transform a problem into a new co-ordinate system
- apply Gauss' divergence theorem to simple problems
- apply Stokes' theorem to simple examples.

Linear optimisation

As a result of learning this material you should be able to
- recognise a linear programming problem in words and formulate it mathematically
- represent the feasible region graphically
- solve a maximisation problem graphically by superimposing lines of equal profit
- carry out a simple sensitivity analysis
- represent and solve graphically a minimisation problem
- explain the term 'redundant constraint'

- understand the meaning and use of slack variables in reformulating a problem
- understand the concept of duality and be able to formulate the dual to a given problem.

The simplex method

As a result of learning this material you should be able to
- convert a linear programming problem into a simplex tableau
- solve a maximisation problem by the simplex method
- interpret the tableau at each stage of the journey round the simplex
- recognise cases of failure
- write down the dual to a linear programming problem
- use the dual problem to solve a minimisation problem.

Non-linear optimisation

As a result of learning this material you should be able to
- solve an unconstrained optimisation problem in two variables
- use information in a physically-based problem to help obtain the solution
- use the method of Lagrange multipliers to solve constrained optimisation problems
- solve practical problems such as minimising surface area for a fixed enclosed volume or minimising enclosed volume for a fixed surface area.

Laplace transforms

As a result of learning this material you should be able to
- use tables to find the Laplace transforms of simple functions
- use the property of linearity to find the Laplace transforms
- use the first shift theorem to find the Laplace transforms
- use the 'multiply by *t*' theorem to find the Laplace transforms
- obtain the transforms of first and second derivatives
- invert a transform using tables and partial fractions
- solve initial-value problems using Laplace transforms
- compare this method of solution with the method of complementary function / particular integral.
- use the unit step function in the definition of functions
- know the Laplace transform of the unit step function
- use the second shift theorem to invert Laplace transforms
- obtain the Laplace transform of a periodic function
- know the Laplace transform of the unit impulse function
- obtain the transfer function of a simple linear time-invariant system
- obtain the impulse response of a simple system
- apply initial-value and final-value theorems
- obtain the frequency response of a simple system.

z transforms

As a result of learning this material you should be able to

- recognise the need to sample continuous-time functions to obtain a discrete-time signal
- obtain the z transforms of simple sequences
- use the linearity and shift properties to obtain z transforms
- know the 'multiply by a^k' and 'multiply by k^n' theorems
- use the initial-value and final-value theorems
- invert a transform using tables and partial fractions
- solve initial-value problems using z transforms
- compare this method of solution with the method Laplace transforms.

Complex functions

As a result of learning this material you should be able to

- define a complex function and an analytic function
- determine the image path of a linear mapping
- determine the image path under the inversion mapping
- determine the image path under a bilinear mapping
- determine the image path under the mapping $w = z^2$
- understand the concept of conformal mapping and know and apply some examples
- verify that a given function satisfies the Cauchy-Riemann conditions
- recognise when complex functions are multi-valued
- define a harmonic function
- find the conjugate to a given harmonic function.

Complex series and contour integration

As a result of learning this material you should be able to

- obtain the Taylor series of simple complex functions
- determine the radius of convergence of such series
- obtain the Laurent series of simple complex functions
- recognise the need for different series in different parts of the complex plane
- understand the terms 'singularity', pole
- find the residue of a complex function at a pole
- understand the concept of a contour integral
- evaluate a contour integral along simple linear paths
- use Cauchy's theorem and Cauchy's integral theorem
- state and use the residue theorem to evaluate definite real integrals

Introduction to partial differential equations

As a result of learning this material you should be able to

- recognise the three main types of second-order linear partial differential equations
- appreciate in outline how each of these types is derived
- state suitable boundary conditions to accompany each type
- understand the nature of the solution of each type of equation.

Solving partial differential equations

As a result of learning this material you should be able to

- understand the main steps in the separation of variables method
- apply the method to the solution of Laplace's equation
- interpret the solution in terms of the physical problem.

3.4.2 Discrete Mathematics

The material in this section covers the basic development of discrete mathematics consequent on the material in Core Level 1.

Number systems

As a result of learning this material you should be able to

- recognise the Peano axioms
- carry out arithmetic operations in the binary system
- carry out arithmetic operations in the hexadecimal system
- use Euclid's algorithm for finding the greatest common divisor

Algebraic operations

As a result of learning this material you should be able to

- understand the notion of a group
- establish the congruence of two numbers modulo *n*
- understand and carry out arithmetic operations in Z_n , especially in Z_2
- carry out arithmetic operations on matrices over Z_2
- understand the Hamming code as an application of the above (any other suitable code will serve just as well).

Recursion and difference equations

As a result of learning this material you should be able to

- define a sequence by a recursive formula
- obtain the general solution of a linear first-order difference equation with constant coefficients
- obtain the particular solution of a linear first-order difference equation with constant coefficients which satisfies suitable given conditions
- obtain the general solution of a linear second-order difference equation with constant coefficients
- obtain the particular solution of a linear second-order difference equation with constant coefficients which satisfies suitable given conditions

Relations

As a result of learning this material you should be able to

- understand the notion of binary relation
- find the composition of two binary relations
- find the inverse of a binary relation
- understand the notion of a ternary relation
- understand the notion of an equivalence relation on a set
- verify whether a given relation is an equivalence relation or not

- understand the notion of a partition on a set
- view an equivalence either as a relation or a partition
- understand the notion of a partial order on a set
- understand the difference between maximal and greatest element, and between minimal and smallest element.

Graphs

As a result of learning this material you should be able to

- recognise an Euler trail in a graph and / or an Euler graph
- recognise a Hamilton cycle (path) in a graph
- find components of connectivity in a graph
- find components of strong connectivity in a directed graph
- find a minimal spanning tree of a given graph.

Algorithms

As a result of learning this material you should be able to

- understand when an algorithm solves a problem
- understand the ‘big O’ notation for functions
- understand the worst case analysis of an algorithm
- understand one of the sorting algorithms
- understand the idea of depth-first search
- understand the idea of breadth-first search
- understand a multi-stage algorithm (for example, finding the shortest path, finding the minimal spanning tree or finding maximal flow)
- understand the notion of a polynomial-time-solvable problem
- understand the notion of an NP problem (as a problem for which it is “easy” to verify an affirmative answer)
- understand the notion of an NP-complete problem (as a hardest problem among NP problems).

3.4.3 Geometry

The material in this section covers the basic development of geometry consequent on the material in Core Level 1.

Helix

As a result of learning this material you should be able to
- recognise the parametric equation of a helix
- derive the main properties of a helix, including the equation of the tangent at a point, slope and pitch.

. **Geometric spaces and transformations**

As a result of learning this material you should be able to
- define Euclidean space and state its general properties
- understand the Cartesian co-ordinate system in the space
- apply the Euler transformations of the co-ordinate system
- understand the polar co-ordinate system in the plane
- understand the cylindrical co-ordinate system in the space
- understand the spherical co-ordinate system in the space
- define Affine space and state its general properties
- understand the general concept of a geometric transformation on a set of points
- understand the terms ‘invariants’ and ‘invariant properties’
- know and use the non-commutativity of the composition of transformations
- understand the group representation of geometric transformatoions
- classify specific groups of geometric transformations with respect to invariants
- derive the matrix form of basic Euclidean transformations
- derive the matrix form of an affine transformation
- calculate coordinates of an image of a point in a geometric transformation
- apply a geometric transformation to a plane figure.

3.4.4 Linear Algebra

The material in this section covers the basic development of linear algebra consequent on the material in Core Level 1.

Matrix methods

As a result of learning this material you should be able to

- define a banded matrix
- recognise the notation for a tri-diagonal matrix
- use the Thomas algorithm for solving a system of equations with a tri-diagonal coefficient matrix
- partition a matrix
- carry out addition and multiplication of suitably-partitioned matrices
- find the inverse of a matrix in partitioned form.

Eigenvalue problems

As a result of learning this material you should be able to

- interpret eigenvectors and eigenvalues of a matrix in terms of the transformation it represents
- convert a transformation into a matrix eigenvalue problem
- find the eigenvalues and eigenvectors of 2x2 and 3x3 matrices algebraically
- determine the modal matrix for a given matrix
- reduce a matrix to diagonal form
- reduce a matrix to Jordan form
- state the Cayley-Hamilton theorem and use it to find powers and the inverse of a matrix
- understand a simple numerical method for finding the eigenvectors of a matrix
- use appropriate software to compute the eigenvalues and eigenvectors of a matrix
- apply eigenvalues and eigenvectors to the solution of systems of linear difference and differential equations
- understand how a problem in oscillatory motion can lead to an eigenvalue problem
- interpret the eigenvalues and eigenvectors in terms of the motion
- define a quadratic form and determine its nature using eigenvalues.

3.4.5 Statistics and Probability

The material in this section covers the basic development of statistics and probability consequent on the material in Core Level 1.

One-dimensional random variables

As a result of learning this material you should be able to
- compare empirical and theoretical distributions
- apply the exponential distribution to simple problems
- apply the normal distribution to simple problems
- apply the Weibull distribution to simple problems
- apply the gamma distribution to simple problems.

Two-dimensional random variables

As a result of learning this material you should be able to
- understand the concept of a joint distribution
- understand the terms ‘joint density function’, ‘marginal distribution functions’
- define independence of two random variables
- solve problems involving linear combinations of random variables
- determine the covariance of two random variables
- determine the correlation of two random variables.

Small sample statistics

As a result of learning this material you should be able to
- realise that the normal distribution is not reliable when used with small samples
- use tables of the t-distribution
- solve problems involving small-sample means using the t-distribution
- use tables of the F-distribution
- use pooling of variances where appropriate
- use the method of pairing where appropriate.

Small sample statistics: chi-square tests

As a result of learning this material you should be able to
- use tables for chi-squared distributions
- decide on the number of degrees of freedom appropriate to a particular problem
- use the chi-square distribution in tests of independence (contingency tables)
- use the chi-square distribution in tests of goodness of fit.

Analysis of variance

As a result of learning this material you should be able to
- set up the information for a one-way analysis of variance
- interpret the ANOVA table
- solve a problem using one-way analysis of variance
- set up the information for a two-way analysis of variance
- interpret the ANOVA table
- solve a problem using two-way analysis of variance.

Simple linear regression

As a result of learning this material you should be able to
- derive the equation of the line of best fit to a set of data pairs
- calculate the correlation coefficient
- place confidence intervals around the estimates of slope and intercept
- place confidence intervals around values estimated from the regression line
- carry out an analysis of variance to test goodness of fit of the regression line
- interpret the results of the tests in terms of the original data
- describe the relationship between linear regression and least squares fitting.

Multiple linear regression and design of experiments

As a result of learning this material you should be able to
- understand the ideas involved in a multiple regression analysis
- appreciate the importance of experimental design
- recognise simple statistical designs.

3.5 Level 3

This level is the one at which the mathematical techniques covered should be applied to a range of problems encountered in industry by practising engineers. These advanced methods build on the foundations laid by Levels 1 and 2 of the curriculum. It is quite possible that much of this material will be taught not within the context of dedicated mathematical units but as part of units on the engineering topics to which they directly apply. It is expected that significant use will be made of industry standard mathematical software tools. The specialised nature of these techniques and the importance of their application in an engineering setting makes detailed learning outcomes (as given for the other levels of the curriculum) less straightforward to define. For this reason only a list of general topic headings will be given. This material will be taught only towards the end of a degree programme.

2.1.1 Analysis and calculus

Numerical solution of ordinary differential equations
Fourier analysis
Solution of partial differential equations, including the use of Fourier series
Fourier transforms
Finite element method

2.1.2 Discrete mathematics

Combinatorics
Graph theory
Algebraic structures
Lattices and Boolean algebra
Grammars and languages

3.5.3 Geometry

Differential geometry
Geometric modelling of curves and surfaces
Geometric methods in solid modelling
Non-Euclidean geometry
Computer geometry
Fractal geometry
Geometric core of Computer Graphics

2. Linear Algebra

Matrix decomposition
Further numerical methods

3.5.5 Statistics and probability

Stochastic processes
Statistical quality control
Reliability
Experimental design

Queueing theory and discrete simulation
Filtering and control
Markov processes and renewal theory
Statistical inference
Multivariate analysis

Other subjects

Chaos theory
Fuzzy mathematics

Underlying the curriculum detailed in Chapter 3 is the view that engineers need not be experts in mathematics per se, but their education and development should allow them to cope with the future demands made upon them. Whilst there is no place for unnecessary rigour there must be a solid mathematical base on which they can build. The learning outcomes of their education which are relevant to mathematics can be expressed by stating that during their professional careers Engineers should be able to:

- understand the mathematical development of theory in areas appropriate to their discipline
- solve problems in their discipline using mathematics where appropriate
- assess the best methods for analysis, be they experimental, analytical or computer-based, in any given situation
- develop analytical methods where appropriate
- use intelligently advanced software tools for design and analysis, (for example, be able to recognise the limits of software tools and be able to recognise mistakes in the analysis process)
- understand and generate simple test cases to prove that software works correctly for these situations
- use mathematical tools to report the results of their work.

For each particular engineering discipline, learning outcomes that are more specific could well be developed for each of these items.

4.1 Entry issues

Many European institutions of higher education are faced with the prospect of an increasingly heterogeneous input to their engineering courses, especially with regard to the mathematical ability of the students. The problems that follow are discussed in a report published by the Institute of Mathematics and its Applications [10].

"Universities have not been unaware of these problems and steps have been taken to address them. One option is to reduce syllabus content, or to replace some of the harder material with more revision (or, for some students, vision!) of lower level work. This may help the weaker students. However, if more advanced material has to be removed to make space for this revision, it disadvantages the more able, making them less well prepared for the mathematical demands of the more advanced and more analytical parts of their engineering study. Many universities have followed a programme of gradual modification of content, but this modification has rarely been radical and has often been seen as chasing a moving target.

"Some universities have developed additional units of study for the weaker students. However, this option is not open to all relevant university departments as many are constrained by modular schemes which do not have sufficient flexibility to require (or even to permit) students to attend more than a specified number of modules.

"The students who need the most help are in a very difficult situation. Their mathematical weakness means that they need to spend a significant amount of time either in studying an extra module or in the mathematics support centre to receive help in remedying the deficiencies of their mathematical background. However, because of these weaknesses they find many of their other engineering subjects difficult and so need to spend extra time trying to keep up in these subjects. As a consequence they tend to view any extra mathematics they undertake as a way of dealing with isolated problems and not as a means of constructing a coherent mathematical knowledge base."

What is clear is that the foundations of the studcnts' mathematical education must be laid carefully and this takes time. There are no quick fixes.

4.2 Teaching issues

The question "Who should teach the mathematics to engineering undergraduate mathematics?" has a long history. The key points are that the teacher must have a wide knowledge of mathematics and its applications in the relevant engineering discipline, and be aware of the changes taking place in secondary education - and their effects.

One danger that is not too far away is the 'just-in-time' philosophy that some engineers are advocating for the teaching of mathematical topics. This could have disastrous consequences. Mathematics is a hierarchical subject and the idea that it can be learned in a piecemeal fashion is not sensible (Just as fluency in a foreign language does not come by learning parrot-fashion a collection of useful phrases.). There is a need to construct a coherent knowledge base with connections between topics rather than a collection of seemingly isolated methods. The 'just-in-time' approach is likely to promote mathematical thinking in engineers at the level of 'problems of this type are solved by method A'. The aim should be the development of an overview of the coherence, power and general applicability of mathematics as a means of analysis, a problem-solving tool and a language of exact scientific communication

There is continuing pressure in many countries to reduce the number of hours devoted to mathematics within engineering degree courses. Whilst there is a recognition that there has been a decline in the mathematics capability of entrants to undergraduate degree courses there has not been a corresponding reduction in the expectations of the students from many engineering colleagues. If anything, more time should be allowed for mathematics, not less. As we have stressed, the foundations of mathematics need to be laid carefully and this requires time. If these foundations have not been well laid then there will be problems for the student subsequently. What pre-university education has failed to deliver must be dealt with at the university itself. It is unreasonable to expect a student to acquire these foundations at the same time as being required to operate at the next level up.

Another issue concerns the teaching of numerical methods. There is considerable benefit from the integration of numerical methods with their analytical counterparts. For example, when introducing ordinary differential equations the discussion of Euler's method as a contrast to simple analytical methods provides a more problem-oriented approach. This enhances a student's appreciation of the range of mathematical tools available for tackling a particular problem. Indeed, for many practising engineers numerical methods are frequently much more important than analytical ones.

4.3 Communication

The need for students to be able to communicate clearly and effectively the results of mathematical investigations is self-evident, yet more emphasis should be placed on this aspect. Equally important is the need for students to be able to 'read' mathematics accurately and intelligently. Some recognition of this could be included in the student's assessment. Mathematics is widely regarded as the language of scientific and technical communication; therefore the need to 'read' and 'write' mathematics correctly is clear. It is not sufficient to 'get the right answer': engineers should be able to communicate succinctly and unambiguously to others the process by which that answer was reached.

4.4 Modelling and mathematics

The ability to formulate a mathematical model of a given physical situation, to solve the model, interpret the solution and refine the model is a key aspect of the mathematical development of an engineer. The need to make initial assumptions and possibly relax some of them in order to obtain a more representative model is an integral part of the process.

Much has been written about the teaching of mathematical modelling. Here we merely outline some main points.

- It is important that the exposition of the modelling process should be introduced as early in the curriculum as is reasonable.
- The first models used should be simple, so that the process is not obscured by the complexity of the problem, by concepts in engineering not yet encountered and by notation with which the student is unfamiliar.
- The mathematics used in the first models used should be straightforward.
- The models must be realistic.
- The physical situation should be one to which the model has been applied in practice. It is of little value applying a mathematical model to a situation to which it has never been applied, simply to make a pedagogic point.
- Models should be introduced sparingly.

4.5 Assessment

There is merit in having a clear strategy on the assessment of the achievement of students as regards the learning outcomes listed in Chapter 3. Assessment must be effective, fair and understood by those being assessed. Many methods of assessment are currently being used and it is wise to keep the procedures under constant review. In an overview paper Mustoe [11] took a wide sweep of the issues in, and the methods of, assessment of mathematics for engineering undergraduates. Particular attention was paid to the written examination and the multiple-choice test, since these are well established and widely used. There were a number of concerns. as to whether even these methods met the objectives of assessment.

4.6 Continuing Education

Modern engineers must undergo many formal, or informal, programmes of continuing education in order to keep up to date with developments in their subject. This requirement is likely to be of even greater importance as technology advances at an ever-increasing pace. A decade ago for example, control theory, reliability engineering and a structured approach to experimental design were presenting challenges to those already in the engineering profession.

The Core Curriculum has been designed to provide a sound basic mathematical education. From this base an engineer should be able to undertake successfully further study of mathematics as changing career demands necessitate a development of skills. The more widely the Core Curriculum is adopted the greater flexibility it offers the engineer in choosing where to undertake such further study.

Chapter 5 Conclusions

The SEFI Mathematics Working Group has prepared this document as a contribution to the cause of educating professional engineers for change in the twenty-first century. We believe that many benefits will flow from the widespread adoption of this Core Curriculum, not least of which is the potential flexibility it offers to students wishing to transfer between universities.

The Working Group believes in international co-operation and the mutual recognition of engineering qualifications; it is hoped that the Core Curriculum has a contribution to make in this area.

We urge all heads of engineering departments to give our proposals serious consideration.

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- to contribute to the development and the improvement of engineering education and of the position of the engineering professionals
- to provide appropriate services and to promote information about engineering education
- to improve communication and exchanges between teachers, researchers and students in the various European countries
- to develop co-operation between educational engineering institutions and establishments of higher technical education
- to promote co-operation between industry and those engaged in engineering education
- to act as an interlocutor between its members and other societies or organisations
- to promote the European dimension in higher engineering education

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- Accreditation and Engineering Education
- Engineering Education and Research
- SEFI and the Bologna Declaration
- EJEE Committee