Developing Learning and Teaching in Engineering Mathematics with and without Technology

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INTRODUCTION

University teachers of mathematics have begun to observe that nowadays new students when beginning their studies do not have as good a mathematical proficiency as before. The phenomenon has been noticed in all western countries during recent decades [1, 2]. What shall we do? We think that there are at least two available courses of action for improved learning results in university mathematics: 1) to identify as soon as possible the students who have an insufficient knowledge base in mathematics, and to begin remedial instruction for them, and 2) to develop mathematics learning environments both with and without technology.

The aim of this paper is to describe how Tampere University of Technology (TUT) has developed learning environments in mathematics during the last decade. We focus in the paper on two cases: 1) a multisemiotic approach to mathematical concepts and procedures, and 2) computer aided assessment and learning systems.

The first case consists of developing studies in mathematical exercises in which new kinds of problem-solving have been constructed. In the second case new students have participated in an ICT –based basic skills test at the beginning of their mathematics studies, to enable them to practice mathematical procedures in solving processes [3]. Electronic and web-based tools make it possible for students to learn independently at any time, and for teachers, offer an effective way to evaluate students' proficiency.

1 THEORETICAL FRAMEWORK

Our objective as mathematics teachers is to develop students' mathematical thinking at all levels of our school system. At university level students are expected to have sufficient mathematical proficiency in order to succeed in their mathematics studies; conceptual understanding and procedural fluency will be particularly emphasized at the beginning of university studies [2].

1.1 Mathematical thinking and mathematical proficiency

We can describe the concept "mathematical thinking" in several ways, depending our point of view: for example, mathematical thinking is an information process monitored by one's metacognition [4]. Kilpatrick, Swafford and Findell [5] constructed a model for student's mathematical proficiency, which could be one way of describing the features of a student's mathematical thinking. It takes account of different types of knowledge (conceptual, procedural and strategic) [4].

Mathematical proficiency is defined as having five components [5]:

1) *conceptual understanding* (comprehension of mathematical concepts, operations, and relations),

2) *procedural fluency* (skill in carrying out procedures flexibly, accurately, efficiently and appropriately),

3) *strategic competence* (ability to formulate, represent, and solve mathematical problems),

4) *adaptive reasoning* (capacity for logical thought, reflection, explanation, and justification) and

5*) productive disposition* (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

We shall concentrate on two of them: conceptual understanding, and procedural fluency. They are seen as traditionally important aims in mathematics education [3, 4, 5].

1.2 A multisemiotic approach to mathematical thinking: languaging mathematics

The importance of natural language in mathematical actions is obvious as well as in science [6, 7, 8]. Results of earlier studies have shown that expressing solutions of mathematical problems in a student's natural language boosts learning in mathematics, develops mathematical understanding (conceptual understanding), changes the student's attitude towards mathematics, and helps a teacher's evaluation work [9]. The use of natural language in the solution process of mathematical problems and in presentation of those solutions helps a student to organize their mathematical thinking for themself and for a peer group [10, 11].

We have chosen the multisemiotic approach [12, 13, 14] to mathematical concepts in both teaching and learning, and see that it is worth using several approaches to give meanings to the mathematical concepts or to algorithms which students are learning.

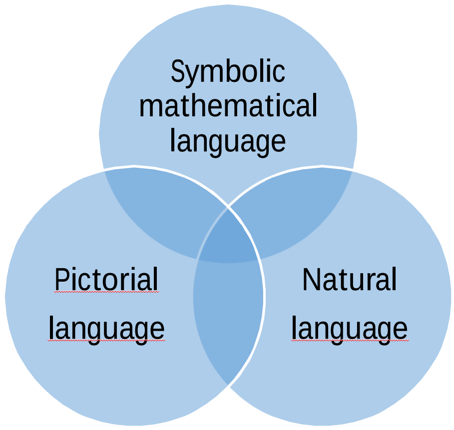


Fig. 1. The three languages which can be used in expressing mathematical thinking (in languaging of mathematics)[2,11,14]

At university and upper-secondary school level we recognize three useful languages in mathematical presentations (*Fig.1.*) and we call them *mathematical symbolic language*, *natural language* and *pictorial language* [11,14]. In a mathematical context this means that a student can express their mathematical thinking either by using mathematical symbols (e.g. numbers, symbols), as well as by natural language (mother tongue) and/or by pictures [7, 11,12]. The main purpose of using several languages in learning activities (e.g. in studying new mathematical concepts and doing exercises) is to develop the student's own meaning making process. We term this process "languaging", a concept which has been used in mathematics and in mother tongue didactics since the 1990's [15, 16]. *Languaging* in mathematics refers to expressing one's mathematical thinking either orally or in writing [11, 14]. Languaging can be seen as a multisemiotic approach to making meanings to mathematical concepts and procedures [12, 14].

Joutsenlahti [11] has identified five basic languaging models for problem solutions (e.g. to word problems). The models are:

1) "*standard*"-model: The whole solution process is presented only by mathematical symbolic language. This is the traditional model.
2) "*storytelling*"-model: The solution process is like a progressive story, in which each step is presented first by natural or pictorial languages, and then by mathematical symbolic language. This is the model used in upper secondary school textbooks.
3) "*roadmap*"- model: The main idea of the solution process is explained at the outset in natural or pictorial language, after which the solver implements it using mathematical symbolic language.
4) "*comment*"- model: The solver makes clarifying comments in natural or pictorial language in parallel with the mathematical solution presentation. Teachers use often this model in their presentations.
5) "*diary*"-model: The solver will use natural or pictorial language only when they need tools to organize their thoughts for progressing in the solution process.

For university students we constructed new kinds of mathematics exercises (compared to the exercises the students were used to do) which lead to the use of natural language in addition to mathematical symbolic language.

1.3 Computer aided assessment and learning

There have been several studies of Computer Aided Assessment (CAA), also known as Computer Aided/Assisted Instruction (CAI). Some studies showed that CAI improved students' achievement, to the extent that the students in the CAI group were able to perform significantly better than students in the control group [17]. However, in other studies the performance was equal between the different groups [18, 19]. A new branch of CAA development is the Intelligent Tutoring Systems (ITS). The difference between CAA and the Intelligent Tutoring System (ITS) is only marginal [17].

At TUT several CAAs are used in mathematics teaching. For more than ten years every new TUT student has participated in the Mathematics Basic Skills Test (BST). The BST is a computer aided test with 16 upper-secondary school mathematics problems to be solved within 45 minutes. The test uses the STACK system [20] making it possible to generate individual problems for each student. Moreover, STACK automatically assesses a student's responses and gives immediate feedback. Thus, all students get their test results right immediately after completing the BST [18]. Fig. 2 shows one example of the BST's exercise.

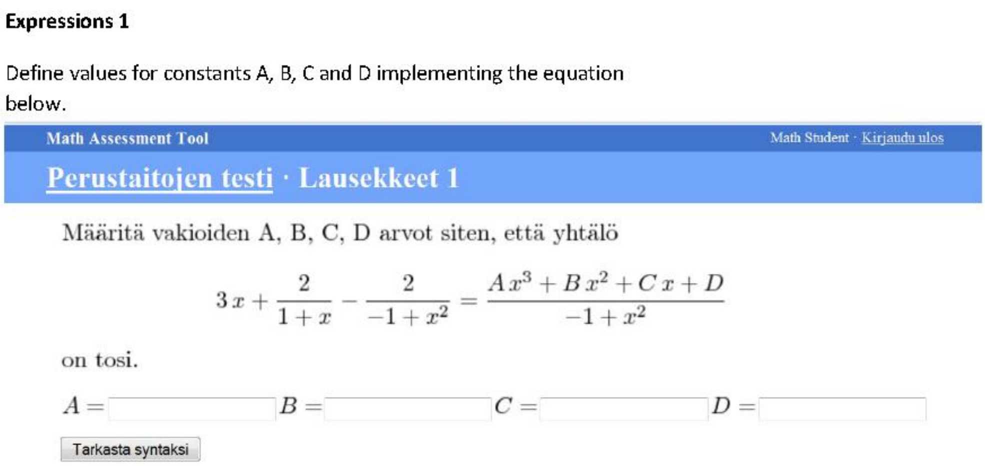


Fig. 2. An example of STACK exercises. This exercise is named "Expressions 1"

In order to pass the BST, a student should be able to complete a set amount out of the 16 assignments within 45 minutes (in the Fall of 2015, the pass limit was 6 for engineering students, 8 for science and mathematics students). Students who failed the test were directed to the Remedial Instruction. The remedial instruction is also carried out using CAI and ITS. [21] Moreover in the basic mathematics courses there are weekly STACK assessments [22].

2 RESEARCH QUESTIONS, DATA AND METHODS

2.1 The research questions

We have two research questions (1. and 2.) in the languaging studies, and one in the ICT-environment (3.):

1. What kind of exercises based on the languaging approach can be designed for mathematics courses at university level?
2. How did the students experience the written languaging in the mathematics exercises?

3. What kind of results and actions has computer aided assessment produced?

The first question leads in fact to a design-based study [23] in which the focus is on the new exercises and on the development process.

2.2 The languaging studies

We have had three interventions in the languaging studies (questions 1 and 2) during the years 2010-2015. The first intervention (Study 1, n=160) at TUT was in 2010. Researchers Kangas, Joutsenlahti and Pohjolainen constructed mathematics exercises, which were based on the five models of written languaging [2, 11]. The second intervention (Study 2, n=116) was in 2012 at TUT and at the University of Turku (UTU). Researchers Sarikka (TUT), Joutsenlahti (TUT), Kangas (TUT) and Harjulehto (UTU) constructed new kinds of prototypes of mathematics exercises, which were based on e.g. coding between the three languages (see Fig.1.) [14, 24]. The third intervention (Study 3, n=182) was at TUT during 2015. Researchers Linnusmäki, Ali-Löytty, Joutsenlahti and Kaarakka continued Study 2 with new exercises. In all the interventions students at TUT solved and gave feedback during the mathematics courses [25].

In the university studies (Study 2 and Study 3) there was a questionnaire which included Likert-scaled statements and open questions. Students had one languaging exercise every week during their mathematics course among traditional exercises. The results were collected both at TUT and at UTU during 2012 autumn semester, and at TUT during 2015.

The method used to analyse the answers to the exercises was qualitative content analysis. The answer to the second problem consists of conclusions from the answers to the Likert-scale questionnaires and open questions. The analysing methods were typical quantitative methods for the questionnaires, and content analysis for the open questions.

By collecting data from students and teachers we were going through the first cycle of the design-based research [23].

2.3 Computer aided assessment

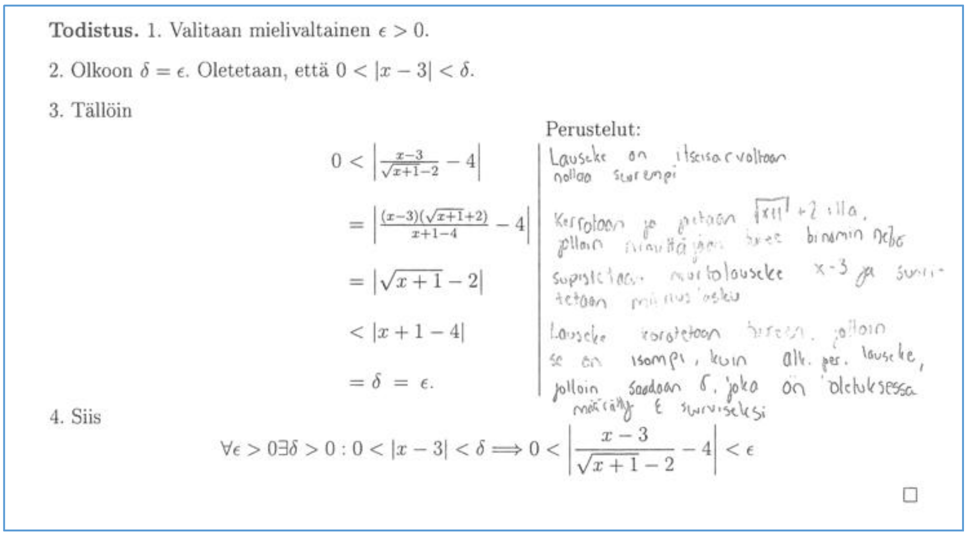
Computer aided assessment can be used to give feedback to the students about their performance on their mathematical (often procedural) skills. The information gathered from technology based learning environments can also be used to improve teaching and teaching methods.

In this paper we present some results obtained from the BST and link the findings to development of teaching. The data is collected from the 16 problems of BST during 2011-2015 and the methods of analysis are statistical. The interpretation of the results is based on the experience of the mathematics teachers.

3 RESULTS

3.1 Languaging based exercises

The second research intervention was in 2012 at TUT. New prototypes of exercises were constructed [11, 14]: 1)”Code-switching”: problem solutions (proofs), which are presented by symbolic mathematical language, are described by natural language (and vice versa), 2) ”Adding missing parts of the solution”: the problem solution is uncompleted, and the student adds the missing parts , 3)” From the solution to a word problem”: the student has a written solution to a problem, to which (s)he will construct a proper word problem, and 4)”Seeking errors”: the student has to find errors (imperfect parts) in the given solution process and correct (complete) them.


Todistus. 1. Valitaan mielivaltainen $\epsilon > 0$.

2. Olkoon $\delta = \epsilon$. Oletetaan, että $0 < |x-3| < \delta$.

3. Tällöin

Perustelut:

$$\begin{aligned} 0 &< \left|\frac{x-3}{\sqrt{x+1}-2} - 4\right| \\ &= \left|\frac{(x-3)(\sqrt{x+1}+2)}{x+1-4} - 4\right| \\ &= |\sqrt{x+1}-2| \\ &< |x+1-4| \\ &= \delta = \epsilon. \end{aligned}$$

4. Siis

$$\forall \epsilon > 0 \exists \delta > 0 : 0 < |x-3| < \delta \Longrightarrow 0 < \left|\frac{x-3}{\sqrt{x+1}-2} - 4\right| < \epsilon$$

□

Fig. 3. An example of a student’s solution to an exercise in which the student codes meanings and expresses arguments by natural language to the proof.

3.2 Students’ experiences of the languaging approach

In the first research intervention (2010) in the first year mathematics courses, there were traditional mathematical problems (n=6) in which students (n=249) were asked explain in their own words how they solved the problems. First of all, 38 (24%) students (n=160) found the languaging exercises clearly negative, and 56 (35%) found them clearly positive. Secondly, most of the students (98, 61%) experienced the use of natural language in solutions of mathematical problems as clearly positive. 20 (13%) students found it clearly negative. The same results were obtained for the Likert-scale statements [2, 14].

The second research intervention was in 2012 at TUT. Here are some examples of the central statements and their acceptance (“I agree”) percentage (n=116) [14, 24]: “Writing my own comments to a math exercise example helps me to understand it better” (81%), “Writing arguments in my own words helps me to understand the exercise better” (83,6%). In the open questions the most commonly appearing theme was “Solving languaging exercises leads to a better understanding of the subject”. It was in 75% of the answers (n=163) [24].

Table 1. Students' experiences about mathematical languaging. Acceptance percentages in the two studies. [24, 25]

Statement	Study 2 (n=116)	Study 3 (n=181)
Such a math exercise which has explanations in natural language, are easier to understand than those that have only math language in them.	89%	79%
I like to explain my math solutions to others	73%	63%
Writing my own comments to a math exercise example helps me to understand it better.	81%	70%
Writing arguments in my own words is easy.	52%	57%
Writing arguments in my own words helps me to understand the exercise better.	84%	77%

The students experienced mathematical languaging very positively in both studies (Table 1). The languaging approach was new for the students and the results of using languaging were encouraged.

3.3 Results of Basic skill tests 2011-2015

The results of the last five years' TUT new student BSTs are shown in Fig 4. The number of participants in these tests during Fall 2010-Fall 2015 are 790, 685, 687, 594, 575 were 652 new students, respectively. From Fig. 4, we can see that the general trend in the students' skills is decreasing, and the differences between the years 2010-2013 and the year 2015 are statistically significant, the p-values of the t-test being less than 0.01. The p-value between the years 2014 and 2015 is $p \approx 0.050$

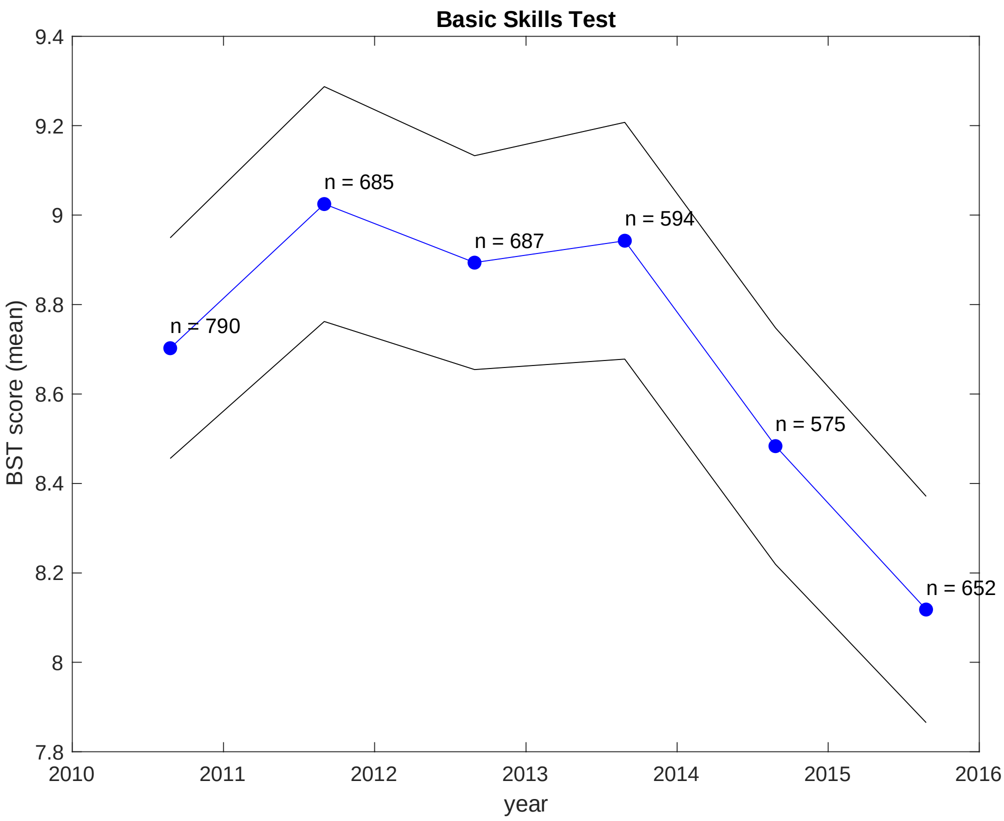


Fig. 4. Mean of the yearly BST scores and 95% confidence interval.

Our results confirm the observation that new students do not have as good a mathematical proficiency as before. For example in Fig. 5 we see that 87% of the students has answered the problem “Numbers 1” and about 61% from all students has correct answer for that problem. The 95 % CI for “Numbers 1” problem on year 2015 is (0.58, 0.65). The p-value is p=0.35730 so the performance is not statistically significantly less.

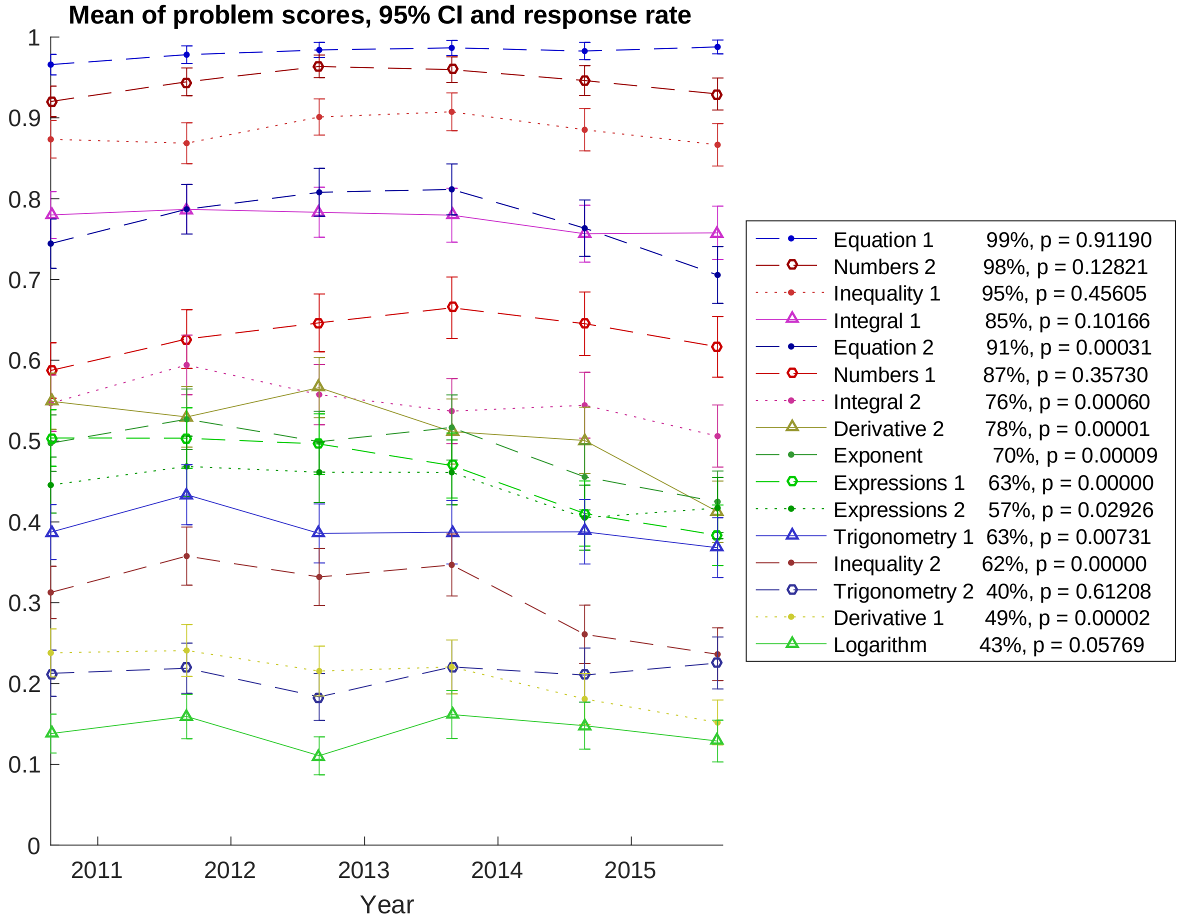


Fig. 5. The mean values of the BST scores for the 16 problems, 95% confidence intervals and response rates.

4 SUMMARY

The BST scores includes at least two indicators: 1) at student level we can guide the students who have insufficient mathematical proficiency (procedural fluency) to remedial instruction, which can be organized in ICT -learning environment (e.g. STACK exercises) and 2) at generation level we can observe that traditional teaching methods at university (and at upper-secondary school) are not any more sufficient, because students’ conceptual understanding has not developed as well as earlier. One solution to the last problem is the languaging approach using e.g. in exercises.

Summa summarum: Future development of university mathematics teaching should take account of: 1) the diversity of students’ mathematical skills and the student’s own opinion about his/her mathematics learning style, motivation, intention etc. , 2) individual guidance and help for students who need it, 3) constructing new kind of exercises which emphasize understanding and 4) taking discussions (using student's own words and expressions) as a meaning making tool for mathematical concepts and algorithms: languaging mathematics. We have to develop learning and teaching in engineering mathematics with and without technology for better conceptual understanding and procedural fluency in mathematics.

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