

Is languaging experienced to improve understanding of structural mechanics?

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INTRODUCTION

In civil engineering education, structural mechanics is a very important part of the studies. Students should understand how structures work before they can design them or calculate their dimensions. Most of the students can do the mechanic calculations but they do not really understand the phenomena.

In structural mechanics, many students try to pass the courses doing as little as possible. Often this means that their exercise solutions have very few visible steps and nothing is written down unless it is absolutely necessary for the reader. Unfortunately, this happens even if it was necessary to write something down in order to someone else to understand the solution process. Many students learn only to imitate similar exercises and do not actually understand what they are doing.

This study was conducted by a teacher developing his own work and it is a part of a broader research on the subject. The aim of the study is to develop the teaching in structural engineering so that students would learn by understanding. It is important that they actually understand the physical concepts and procedures, which are central from the point of view of structural engineering.

The method chosen was languaging, which has given promising results in mathematics education. [1, 2, 3]. Languaging forces students away from minimalistic expression and makes them show what is done and why. In the process students hopefully learn to understand what really happens in the exercises.

1 THEORETICAL FRAMEWORK

1.1 Concepts

In this study, natural language means national languages which usually are students' native languages for example English or Finnish. In this study students had to use Finnish when they did their languaging exercises. Symbolic mathematical language means markings typical to mathematics or mechanics which are unique and logical. Pictorial language means using pictures instead of letters or symbols. Tactile functional language means that one can use for example hands on materials. [4].

1.2 Languaging

In this study, languaging means expressing one's thinking in writing or orally. Students are guided to organize and explain their answers and describe solution processes in their own words or pictures, which is especially important with scientific concepts. When a student has to make meaning of concepts, his/her understanding of the matter increases. Languaging can be utilized in oral situations and written exercises. When a student expresses to others for example the meaning of a concept, he/she has to ponder its main characters and reflect, analyse and improve his/her thinking. This practice allows the teacher to evaluate how students think. This can be helpful for the teacher in pedagogical planning since problematic issues are easier to see. This has been the early application of languaging: to see how someone is thinking so mistakes can be corrected. Now it is seen as a method to learn: one has to structure the issue to oneself before one can explain it to others. [1, 4, 5, 6, 7, 8, 9].

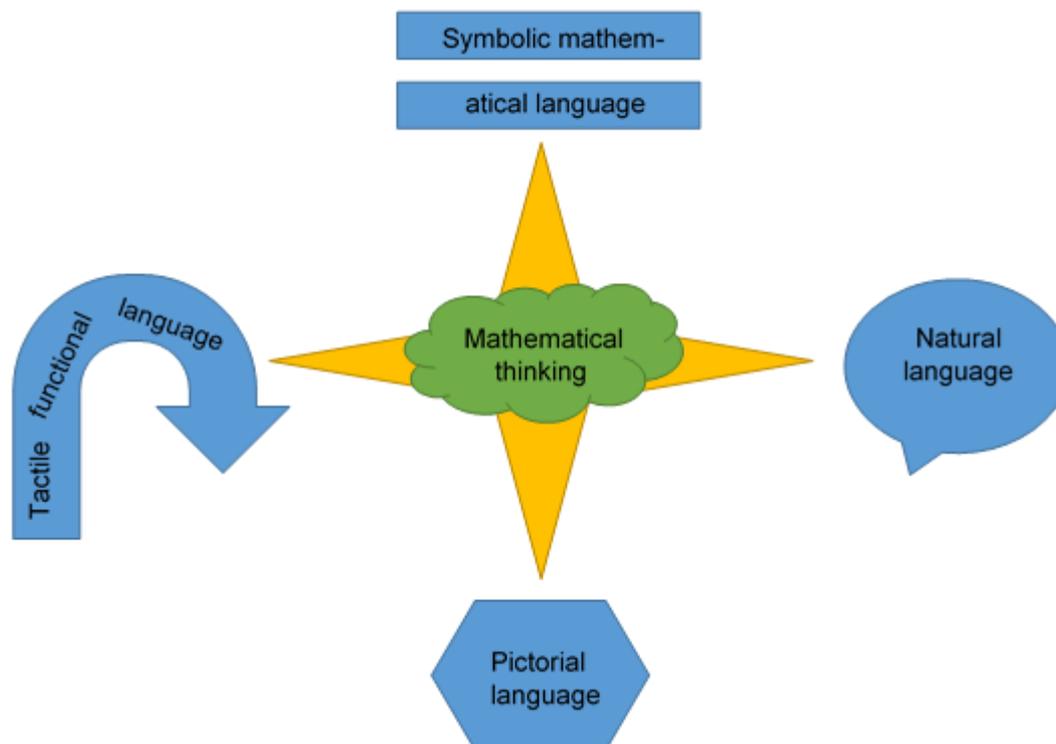


Fig. 1. Expressing mathematical thinking with four languages. [4].

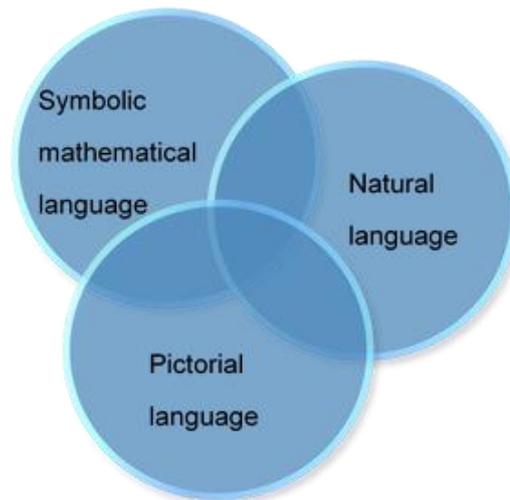


Fig. 2. Three types of mathematical thinking in languaging. [2].

Normally only symbolic mathematical language is used when solving mathematical exercises. In languaging, natural, pictorial and tactile functional languages can also be used to express mathematical thinking as shown in Figure 1. Earlier these languages were seen more consecutive steps. First a child starts to use tactile functional language, then natural language and pictorial language. And finally learns to use symbolic languages. Instead of switching to more advanced language all of these languages can be used simultaneously to improve one's understanding of issues. In this study, the use of languages was for practical reasons limited to these three overlapping languages as presented in Figure 2. [2, 4, 8, 9]

There are five basic languaging models for written problem solutions: 1) Standard -model, where only symbolic language is used. 2) Storytelling -model, where natural/pictorial and symbolic language take turns. 3) Roadmap -model, where natural/pictorial language is used first to explain all the steps and to explain the answer in the end. 4) Comment -model, where symbolic and natural/pictorial language exist side by side in two columns. 5) Diary -model, where natural/pictorial language is used when needed. These models are presented graphically in Figure 3 where SML (blue colour) means symbolic mathematical language, NL (yellow colour) means natural language and PL (also yellow colour) means pictorial language. [1, 2, 7, 10].

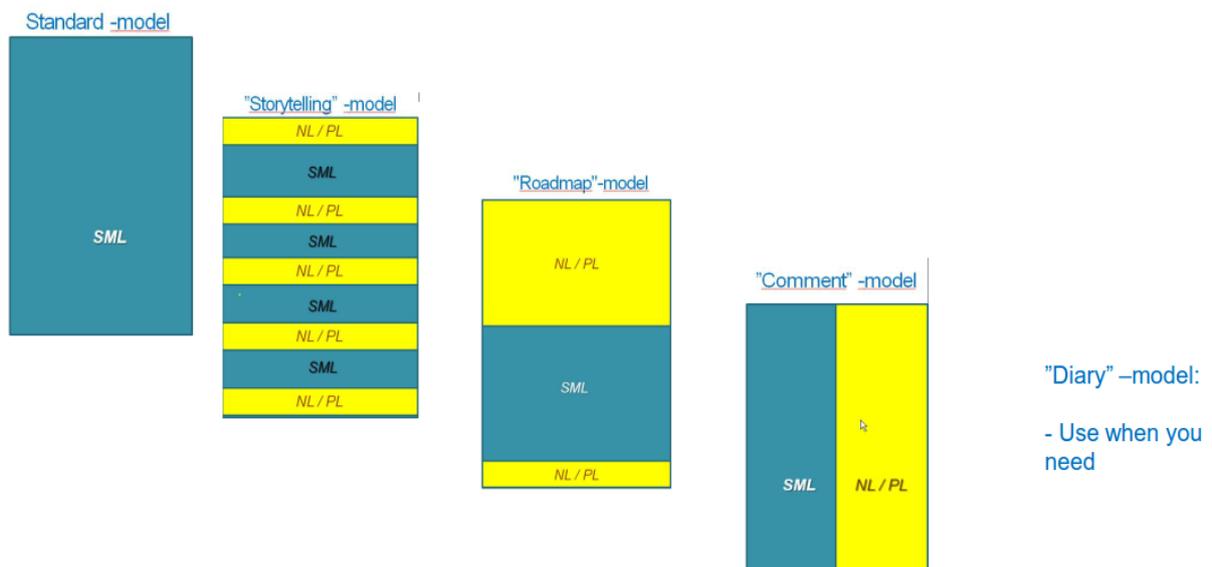
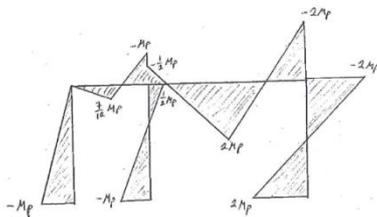


Fig. 3. Five basic languaging models. Blue colour indicates symbolic mathematical language and yellow colour indicates natural or pictorial language [2].

Previous studies have conducted teaching experiments using different types of exercises that require the use of languaging in the solution. Here are eight examples of the types of exercises that require the use of languaging. 1) In “code-switching” the solution to a problem is presented in symbolic mathematical language and the student has to explain in natural/pictorial language what has been done in the solution or vice versa. 2) In “adding missing parts” there are parts missing from the solution and the student has to complete it. 3) In “from a solution to a word problem” the student has to construct a proper question to the problem he/she has the answer to. 4) In “seeking errors” the student has to find errors in a solved problem. 5) In “argumentation of the solution” the student has to explain a ready or self-made solution using different languages. 6) In “data filtering” the student has to find relevant things from the question which includes useless information. 7) In “explaining in your own words” the student has to explain something without using symbolic mathematical language. 8) In “organizing” the student has to put given parts of a solution to a logical order. [1, 2, 7, 10].

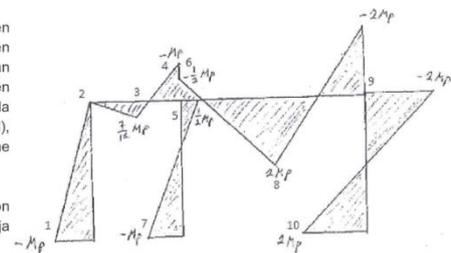
Kielentämistehtävä 3

Selitä omin sanoin mahdollisimman kattavasti mitä voit päätellä alla olevasta momenttikuvioista. Voit halutessasi käyttää myös kuvia selityksesi tukena.



Koska kaikkien pilarien alapäässä on plastisen momentin verran taivutusmomenttia niiden vasemmalla puolella (plastinen nivel), nähdään, että rakenne kaatuu oikealle päin.

Plastisia niveliä on kohdissa 1, 4, 7, 8, 9 ja 10 kuvan 1 mukaisesti.

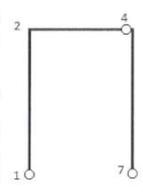


Kuva 1: taivutusmomenttikuvia rajakuormalla

Plastisia niveliä ei ole kuitenkaan vielä tarpeeksi, jotta rakenne sortuisi, sillä nurkkapala 1-2-4 pitää rakennetta pilarin 5-7 kanssa pystyssä (kuva2), joten tässä osassa on pakko olla jokin ylimääräinen nivel. Kuvasta voidaan päätellä, että sen on pakko olla tavallinen nivel pisteessä 2, koska siinä ei ole taivutusmomenttia.

Eri palkkeilla olevien suurimpien plastisten momenttien perusteella voidaan päätellä, että kaikkien 6-9 ja 9-10 plastinen momentti on $2M_p$, ja muiden $1M_p$.

Koska rakenteeseen on syntynyt 6 plastista niveltä, on se alunperin ollut viidesti staattisesti määräämätön. Voidaan päätellä, että jokainen pystypilari on alapäästään jäykästi tuettu, jolloin rakenteessa on 9 tukireaktiota. Se on silloin kuudesti staattisesti määräämätön. Mutta rakenteen nivel pisteessä 2 vähentää hyperstaattisuuden kertalukua yhdellä, joten rakenne on alunperin viidesti staattisesti määräämätön.



Kuva 2: staattisesti määrätty osa 1-2-4-7

Taivutusmomenttikuvion laskemiseen vaaditaan taivutusmomenttitieto yhdeksästä pisteestä. Rakenne on viidesti staattisesti määräämätön. Tällöin alkeismekanismin lukumääräksi saadaan

$$m = s - ns = 9 - 5 = 4.$$

Koska rakenne kaatuu oikealle, on siinä oltava jokin oikealle vaikuttava voima. Voiman on kohdistuttava johonkin vaakasuoraan palkkiin, koska pystysuorissa palkeissa ei näy ”kärkeä” taivutusmomenttikuviossa. Vaakasuorissa palkeissa sen sijaan näkyy kärjet keskellä palkkeja, joten niihin on kohdistuttava pystysuora voima (alaspäin) kohtiin 3 ja 8.

Fig. 4. An example of a languaging exercise where the student has to explain what can be deduced from the given picture.

An example of a languaging exercise is shown in Figure 4. In the left column is the assignment and in the right column is an answer made by a student. The assignment was to explain what can be deduced from the given bending-moment diagram. The student has explained for example where the plastic hinges are, why there has to be a regular hinge and where it is and where point loads will be.

In Finland, languaging has been studied in teaching mathematics both at the upper secondary school and the university level. The results of these studies have been very encouraging. Students have mainly given very positive feedback and found languaging to be a useful, though arduous, method. [1, 2, 3]. The good results in mathematics

suggest that languaging as a method also suits structural mechanics, as it is a mathematical field.

1.3 Objectives on the course

The course, where the study was conducted, was Theory of Plasticity. It is the students' fourth course in structural mechanics. For students studying civil engineering it is compulsory for those who wish to orientate towards structural engineering. To other participants it is voluntary. The main subject in the course was theory of plasticity and its uses in different situations. Students should learn to understand what the difference between elastic and plastic behaviour is and how to use plasticity in calculations. [11, 12].

2 RESEARCH QUESTION, DATA AND METHODS

2.1 Research question

The research task was to develop and try out new kinds of exercises in structural mechanics and the research question was how students experience these exercises. The main point was to map out students' opinions and develop exercises together with the students.

2.2 Data

The data was gathered in Tampere University of Technology in the fall of 2015 during regular teaching of structural mechanics. There were 58 students starting the course and 41 of them did at least one of the new exercises and filled out a questionnaire at the end of the course. The gathered data is identifiable.

There were 6 languaging exercises on the course. They were made so that students were guided to use languaging with the comment -model, where symbolic and natural/pictorial languages exist side by side in two columns. Most of the exercises were combinations of the types presented earlier.

2.3 Methods

In the end-of-course questionnaire, there were 32 Likert scaled and 2 open questions mapping thoughts and attitudes towards languaging, 8 Likert scaled and one open question mapping attitudes towards specific types of languaging exercises and 6 Likert scaled and 1 open question mapping attitudes towards the languaging exercises used on the course. There were also other questions related to other things on the course in the questionnaire.

Open questions were processed with qualitative content analysis and typical quantitative methods were used for the questionnaires.

3 RESULTS

3.1 Students' experiences of the languaging

The students' attitudes towards languaging were mapped with 32 Likert scaled questions. The word languaging was not used in all of the questions but the ideas were expressed verbally. From those questions 13 were selected to show the main ideas. These results are shown in Figure 5 and Table 1.

Students' views on languaging

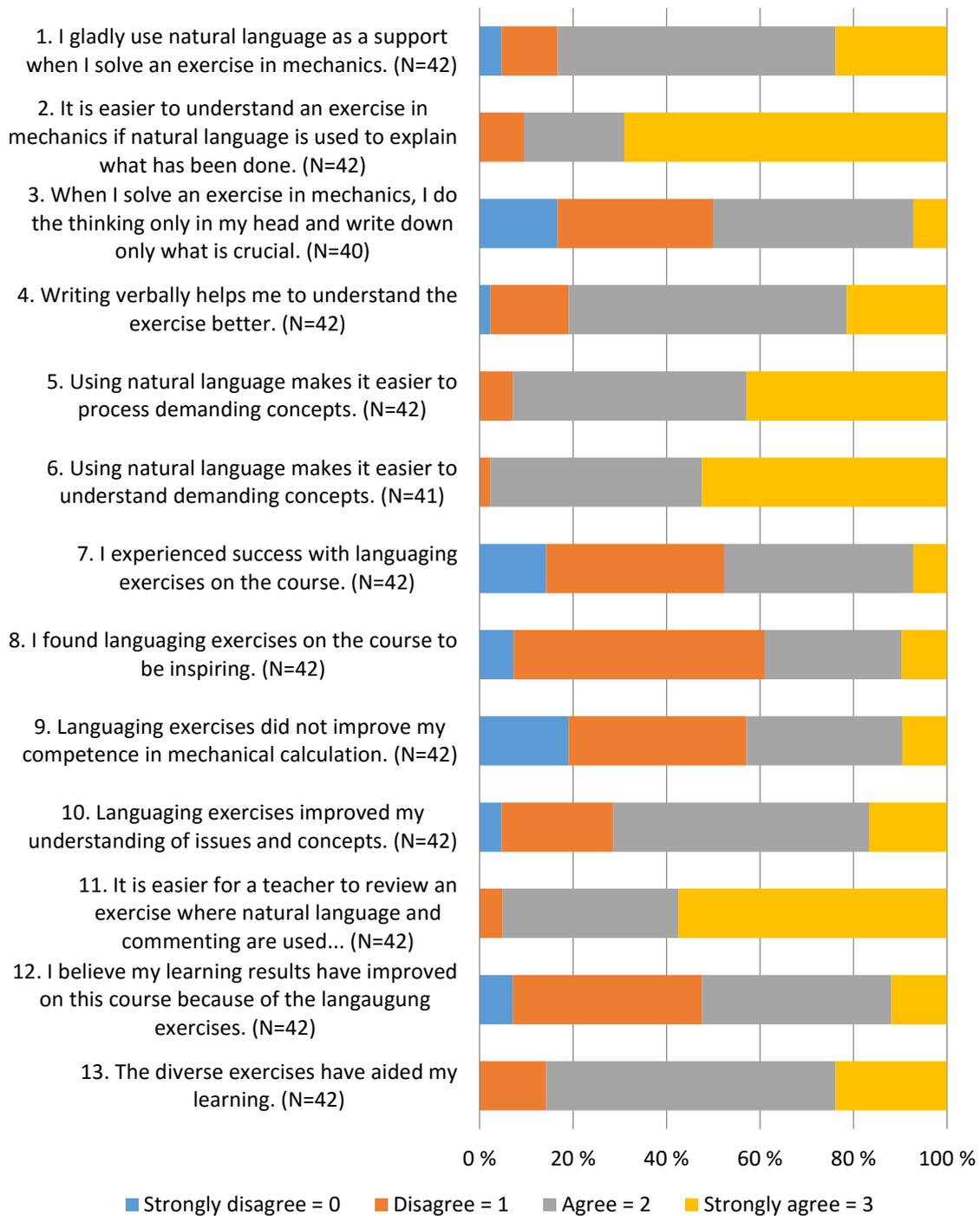


Fig 5. Students' views on languaging

As seen in Figure 5 and Table 1 the results are encouraging. For example, over 90 % of the participants agreed or strongly agreed that it is easier to understand a solution to an exercise when natural language is used side by side with symbolic language. Over 70 % of the participants agreed or strongly agreed that languaging exercises promoted their understanding of issues and concepts. Over 85 % of the participants agreed or strongly agreed that diversification of the exercises has supported their learning. Over 50 % of the participants agreed or strongly agreed that their learning results improved on the course because of the languaging exercises.

Table 1. Students' views on languaging

| Proposition | Agreed (%) | Average | Standard deviation |
|-------------|------------|---------|--------------------|
| 1. | 83,3 | 2,0 | 0,7 |
| 2. | 90,5 | 2,6 | 0,7 |
| 3. | 50,0 | 1,4 | 0,8 |
| 4. | 81,0 | 2,0 | 0,7 |
| 5. | 92,9 | 2,4 | 0,6 |
| 6. | 97,6 | 2,5 | 0,5 |
| 7. | 47,6 | 1,4 | 0,8 |
| 8. | 39,0 | 1,4 | 0,8 |
| 9. | 42,9 | 1,3 | 0,9 |
| 10. | 71,4 | 1,8 | 0,8 |
| 11. | 95,0 | 2,5 | 0,6 |
| 12. | 52,4 | 1,6 | 0,8 |
| 13. | 85,7 | 2,1 | 0,6 |

3.2 Students' views on usefulness of different types of languaging exercises

The students' attitudes towards eight different types of languaging exercises were mapped with Likert scaled questions. The students were emphasized that these are just general types of exercises, no direct match can be found to the exercises on the course. The students had to give their opinion on two different things: how useful the exercise type is to subject's mechanical learning and how useful the exercise type is to understanding the subject. The eight types of languaging exercises were explained as in Table 2.

Table 2. Types of languaging exercises

| | |
|-----|--|
| T1. | Code-switching: so called translation between different fields of mathematical languages so that the essential content does not change. For example from a formula or a picture to natural language. |
| T2. | Adding missing parts: Completing essential parts to an existing solution. |
| T3. | Seeking errors: finding errors in a given solution. |
| T4. | From a solution to a word problem: deducing from the answer what was asked i.e. assignment that the solution fits to. |
| T5. | Argumentation of the solution: explaining a ready or self-made solution using different mathematical languages. |
| T6. | Data filtering: finding essential things for a solution from the assignment. |
| T7. | Explaining in your own words: explaining something without using symbolic mathematical language. |
| T8. | Organizing: putting given parts of a solution to a logical order. |

The results are in Table 3, depending on the question 40–42 students answered. The usefulness of the exercise types to understanding was particularly interesting in the scope of this study. All but one type were thought to be very useful or somewhat useful by over 70 % of the participants. For example over 95 % of the participants thought that the code-switching languaging exercise is somewhat useful or very useful to understanding the issue.

Table 3. Usefulness of the languaging exercises

| Usefulness to subject's mechanical learning | | | | Usefulness to subject's understanding | | | |
|---|------------|---------|--------------------|---------------------------------------|------------|---------|--------------------|
| Exercise type | Useful (%) | Average | Standard deviation | Exercise type | Useful (%) | Average | Standard deviation |
| T1. (N=40) | 77,5 | 1,9 | 0,7 | T1. (N=41) | 95,1 | 2,4 | 0,6 |
| T2. (N=40) | 80,0 | 2,1 | 0,8 | T2. (N=41) | 78,0 | 2,0 | 0,7 |
| T3. (N=41) | 36,6 | 1,3 | 0,9 | T3. (N=42) | 40,5 | 1,3 | 1,0 |
| T4. (N=41) | 43,9 | 1,4 | 0,9 | T4. (N=42) | 73,8 | 1,9 | 0,9 |
| T5. (N=40) | 80,0 | 2,1 | 0,9 | T5. (N=41) | 87,8 | 2,3 | 0,8 |
| T6. (N=41) | 58,5 | 1,7 | 0,9 | T6. (N=42) | 85,7 | 2,1 | 0,8 |
| T7. (N=40) | 72,5 | 1,9 | 0,8 | T7. (N=41) | 90,2 | 2,4 | 0,8 |
| T8. (N=40) | 77,5 | 2,1 | 0,8 | T8. (N=41) | 82,9 | 2,1 | 0,7 |

3.3 Languaging in exercises

In this study there were 6 languaging exercises. The exercises were made so that the students were guided to use languaging with comment model. Most of them were combinations of the types presented in the section 1.2 Languaging. The first one was a combination of “code-switching”, “adding missing parts”, “from a solution to a word problem” and “argumentation of the solution”. The second one was a combination of “code-switching”, “adding missing parts”, “seeking errors” and “argumentation of the solution”. The third one was a “code-switching” exercise with pictorial language. The fourth one was a combination of “code-switching”, “adding missing parts”, “argumentation of the solution” and “organizing”. The fifth one was a “data filtering” exercise. The sixth one was a combination of “code-switching” and “argumentation of the solution”.

Kielentämistehtävä 1

Alla on vasemmassa sarakkeessa esitetty tehtävän ratkaisun matemaattiset lausekkeet.

A) Mitä ratkaisussa on tehty? Kirjoita oikeaan sarakkeeseen mahdollisimman selkeästi ja omin sanoin ilmaistuna, mitä tehtävässä on tehty.

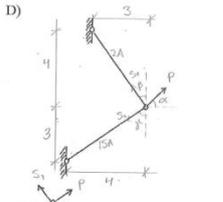
B) Kirjoita puuttuvat välivaiheet.

C) Kirjoita tehtävän vastaus.

D) Täydennä ratkaisun perusteella tehtävän kuva.

E) Kirjoita myös ratkaisuun sopiva tehtävänanto.

Jos tila ei riitä, voit jatkaa toiselle paperille.



D)

B)

$$\rightarrow P \cos \alpha - S_1 \sin \beta - S_2 \sin \gamma = 0$$

$$\uparrow P \sin \alpha + S_1 \cos \beta - S_2 \cos \gamma = 0$$

ratkaisu

$$\uparrow P \cos 45^\circ + \frac{4}{5} S_1 - \frac{3}{5} S_2 = 0$$

$$\rightarrow P \sin 45^\circ - \frac{3}{5} S_1 - \frac{4}{5} S_2 = 0$$

$$\uparrow \frac{1}{\sqrt{2}} P + \frac{4}{5} S_1 - \frac{3}{5} S_2 = 0 \quad || \cdot 3$$

$$\rightarrow \frac{1}{\sqrt{2}} P - \frac{3}{5} S_1 - \frac{4}{5} S_2 = 0 \quad || \cdot 4$$

$$\uparrow \frac{3}{\sqrt{2}} P + \frac{12}{5} S_1 - \frac{3}{5} S_2 = 0$$

$$\rightarrow + \frac{4}{\sqrt{2}} P - \frac{12}{5} S_1 - \frac{16}{5} S_2 = 0$$

$$\frac{7}{\sqrt{2}} P + 0 - \frac{25}{5} S_2 = 0$$

E) Laske staattisesti määrätyn ristikon rajakuorma. $\alpha = 45^\circ$, $A_1 = 2A$, $A_2 = 15A$, ruuut kuvassa. Sauvat ovat rakenneterästä.

A) & B)

Kuormatettavasta nivelekkeisestä pisteestä on otettu tasapainoyhtälöt y- ja y-suunnissa. Summan tulee olla pisteessä 0, koska rakenne on tasapainossa. P on vaakasuuntainen kuormitus (veto) ja S_1 ylemmän sauvan jännitys ja S_2 alemman sauvan jännitys. Komponeerit on käsitelty trigonometrian avulla.

Rakenteen mittojen ja annettujen α -kulman avulla lausekkeet on annettu x-suorintaan yhtiös on kerrottu puolittain 3:lla ja y-suorintaan 4:llä, jotta yhdistäminen ratkaisemisen helpottuu. Yhtälöt on summattu alkuun, jolloin muuttujat S_1 supistuu ja saadaan lauseketta S_2 :n riippuvuus A:hen.

B)

$$\Rightarrow \frac{7}{\sqrt{2}} P - 5 S_2 = 0 \Rightarrow S_2 = \frac{7\sqrt{2}}{10} P$$

$$\frac{1}{\sqrt{2}} P + \frac{4}{5} S_1 - \frac{3}{5} \left(\frac{7\sqrt{2}}{10} P \right) = 0$$

$$\Rightarrow S_1 = -\frac{\sqrt{2}}{10} P$$

$$\delta_1 = \frac{S_1}{A_1} = \frac{-\frac{\sqrt{2}}{10} P}{2A} = -0,071 \frac{P}{A}$$

$$\delta_2 = \frac{S_2}{A_2} = \frac{\frac{7\sqrt{2}}{10} P}{15A} = 0,066 \frac{P}{A}$$

$$\delta = 0,071 \frac{P}{A} = \delta_m \text{ muotti}$$

myötökuorma

$$\Rightarrow P_m = \sigma_m \cdot A \cdot \frac{1}{0,071}$$

$$= 14,14 A \sigma_m$$

plastinen rajakuorma

$$\Rightarrow P_p = 14,14 A \sigma_m$$

A) & B)

Saadon ratkaisua S_2 :n riippuvuus P:stä. Siirtämällä tulos laskelman alku jännityksen lausekkeesta, saadaan S_2 :n riippuvuus P:stä.

Lasjetaan sauvojen poikkipinta-alaan kohdistuvat jännitykset. Jännitys asteeseen jatkamalla yhdistetään voima poikkipinta-alaan. Sauvan 1 poikkipinta-ala on $2A$, 2:n sauvan $15A$. Huomataan, että sauvaan 1 muodostuu puristusjännitystä ja sauvaan 2 vetojännitystä.

Koska sauvat ovat materiaalitilan rakenneterästä olt. vetolujuus = puristuslujuus, jolloin sauva 1 on rakenteen heikompi osia.

Kuorman P kasvaessa sauva 1 plastisoituu, kun $\sigma_m = \sigma_p$.

Tämän avulla lausetaaan myötökuorma ja edelleen plastinen rajakuorma.

o) Myötökuormaksi P_m saadaan $14,14 A \sigma_m$, joka on samalla plastinen rajakuorma, koska rakenteen rasitettavimman osan muutos, koko rakenne sortuu.

Fig. 6. First languaging exercise on the course. The students had to complete the picture and calculations and explain what has been done.

The students returned all together 219 solutions to languaging exercises. In Figures 6 and 8 are two examples of a solution to a languaging exercise made by a student. Solutions are shown in two paragraphs to save space.

Figure 6 is an example of a solution to the first languaging exercise on the course. It was a combination of “code-switching”, “adding missing parts”, “from a solution to a word problem” and “argumentation of the solution”. The parts that were already in the paper given to the students are black and the markings of the student are grey. As seen from the example, the students had to complete the picture and some parts of the calculations as well as give verbal explanation on the right side of the paper. They had to compose a proper assignment to the solution which is marked E) in the given example. They also had to compose a proper answer to the assignment which is marked C) in the given example. The given solution is to calculate the plastic limit load of the given truss.

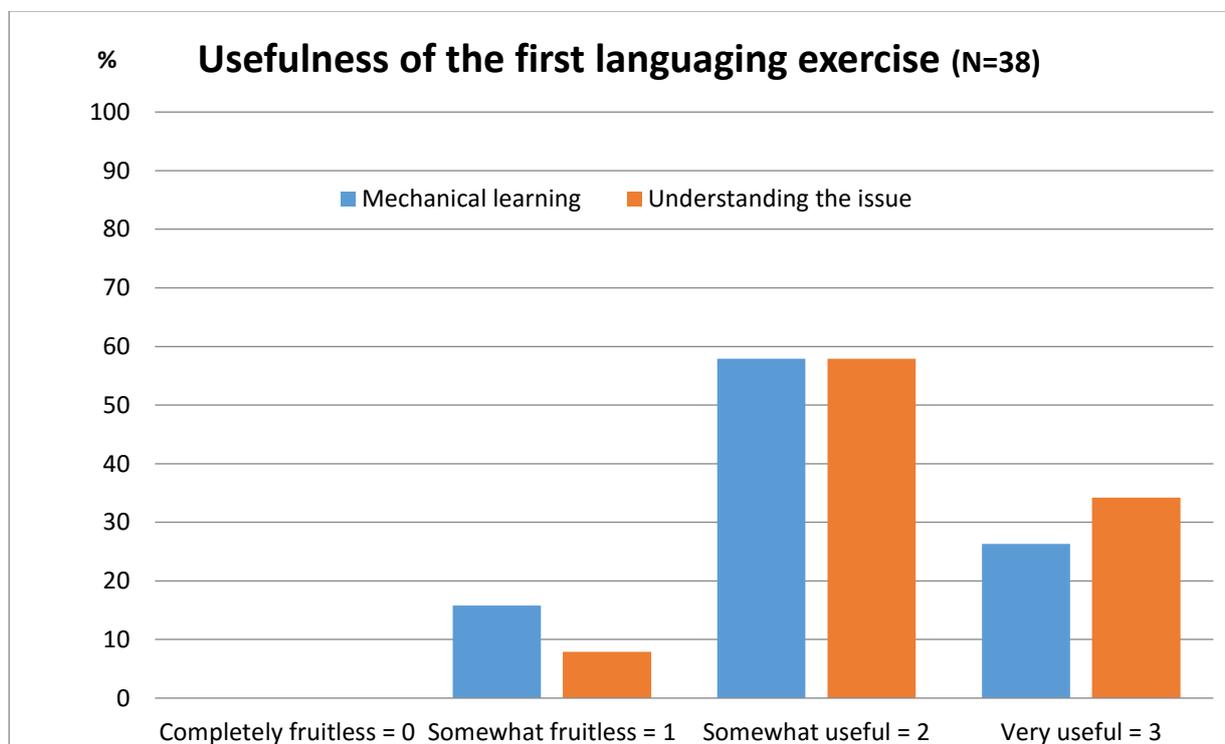


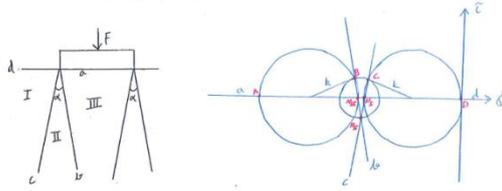
Fig. 7. Usefulness of the first languaging exercise on the course.

Figure 7 shows how useful the students found this first languaging exercise. Usefulness is divided into two parts: how useful it was to mechanical learning of the subject and how useful it was to understanding of the subject. 44 students returned a solution to this exercise and 38 students answered to the question in the Figure 7. On the question “How useful it was to mechanical learning of the subject” average of the answers was 2,1 and the standard deviation 0,6. On the question “How useful it was to understanding of the subject” average of the answers was 2,3 and the standard deviation 0,6.

Kielentämistä
tehtävä 6

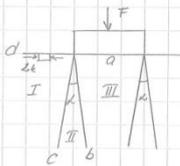
Alla on tehtävänanto ja siihen liittyvä graafinen ratkaisu. Selvitä omin sanoin hyvin selkeästi, miten ratkaisussa käytetty kuva on saatu aikaan ja miten ratkaisu on saatu kuvasta. Halutessasi voit laskea annetusta kuvasta ratkaisun omalla tavallasi. Muista myös siinä tapauksessa selvittää miten ratkaisu on kuvasta saatu.

Keskeinen pistekuorma F vaikuttaa jäykän leimasimen välityksellä kokoonpuristumattomaan ideaaliplastiseen alustaan, jonka leikkausmyötöraja $\tau_m = k$. Leimasimen leveys on b. Määritä myötökourman alaraja F¹ käyttäen kuvan mukaista symmetristä jännitystilakentän vyöhykejakoa, kun $\alpha = 20^\circ$.

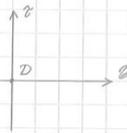


$$F = b \cdot 2 \cdot \left(k \cdot k \cdot \cos 20^\circ + \frac{k \cdot \sin 20^\circ}{\tan 70^\circ} \right)$$

Muutetaan vyöhykkeet numeroin I, II ja III ja vyöhykkeiden rajat a, b, c ja d.



Koska viiva d on ilmaa vasten, ei pisteessä D ole jännitystä. Piste D piirretään origoon z-z-koordinaatissa.



Koska alusta on ideaaliplastinen, on leikkausmyötöraja plastisessa alueessa.

Mahin ympyrän säde on tällöin mahdollisimman eli $r = r_m = k$. Pääjännitykset alueessa I ovat 0 ja $-\sigma_k$ (Pirttavaheet on piirretty aurocadilla erikseen liittämisen). Piirretään k säteen ympyrä D-pisteen ja $-\sigma_k$ kautta, vaihe 1). Ympyrän I napa N_1 määritetään pirttamalla D-pisteen kautta d-tason suuntainen suora, joka leikkaa ympyrän pisteessä N_1 , vaihe 2).

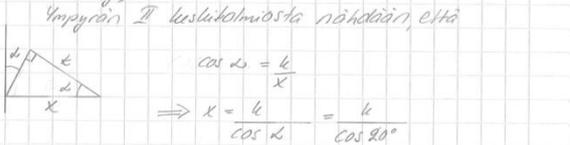
Tämä piste esittää myös jännitystilac suunnassa c eli nyt myös c-suuntainen suora kulkee N_1 :n kautta ja leikkaa ympyrän I pisteessä C, vaihe 3). Ympyrän II säde määräytyy ehdosta

$$k \sin 2\varphi = r_{II} \cos 2\varphi$$

$$\Rightarrow r_{II} = k \tan 2\varphi, \text{ jossa } 2\varphi = \alpha - 20^\circ$$

Toinen sanaan pisteestä C piirretään ympyrän I sädellä k kohtisuora viiva, tällöin akselin z ja viivan välinen kulma on α . z-akselin leikkauspisteessä on ympyrän II keskipiste. Piirretään ympyrä II, vaihe 4). Edetään suoraa c pitkin siten, että saannetaan ympyrän II leikkauspiste. Tällä kohtaa on ympyrän II napa N_2 . Piirretään tällöin N_2 kautta b-tason suuntainen suora, suoran ja ympyrän II leikkauskohtaan merkitään piste B, vaihe 5).

Kulma α on pyörysuoraa akselin nähden symmetrinen, joten ympyrä III voidaan piirtää symmetrisesti ympyrän II keskipisteen suhteen. Eli ympyrän III säde on k ja keskipiste on suoralla z. Ympyrän napa N_3 on ympyrän III kehän ja suoran b leikkauspisteessä. Tästä pisteestä siirrytään a-suoran suuntaisella lunnas leikataan ympyrän III lehti pisteeseen A, vaihe 6).

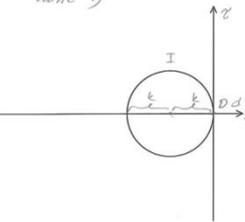


Jolloin symmetrisenä:

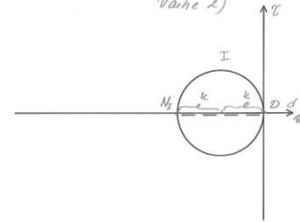
$$\frac{F}{b} = 2k + 2 \cdot \frac{k}{\cos 20^\circ}$$

$$\Rightarrow F = 2kb \left(1 + \frac{1}{\cos 20^\circ} \right)$$

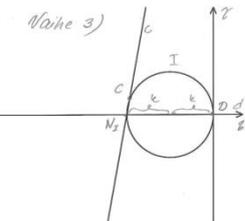
Vaihe 1)



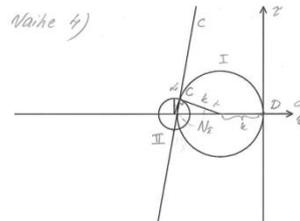
Vaihe 2)



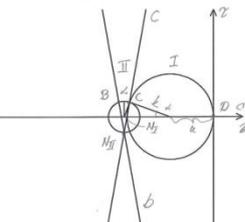
Vaihe 3)



Vaihe 4)



Vaihe 5)



Vaihe 6)

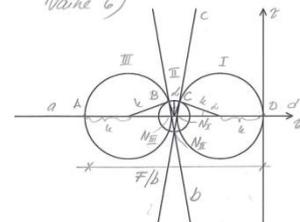


Fig. 8. Last languaging exercise on the course. The students had to explain how the solution picture was constructed and how to get the actual answer from it.

Figure 8 gives an example of a solution to the last languaging exercise on the course. It was a combination of “code-switching” and “argumentation of the solution”. The parts that were already in the paper given to the students are the black, blue and red markings in the top part of the left paragraph. The rest of the markings were made by the student, partly with a computer program. The students were given an assignment and a graphical solution to it. They had to explain how the solution was constructed and how the actual answer was derived from this picture. In the given assignment and

its solution, a rigid item is pressed against incompressible ideally plastic ground and a lower limit to the load is calculated using Mohr circles. The student has explained step by step how Mohr circles are drawn and used pictures to clarify it. Then the student has shown how to get an exact answer from the picture.

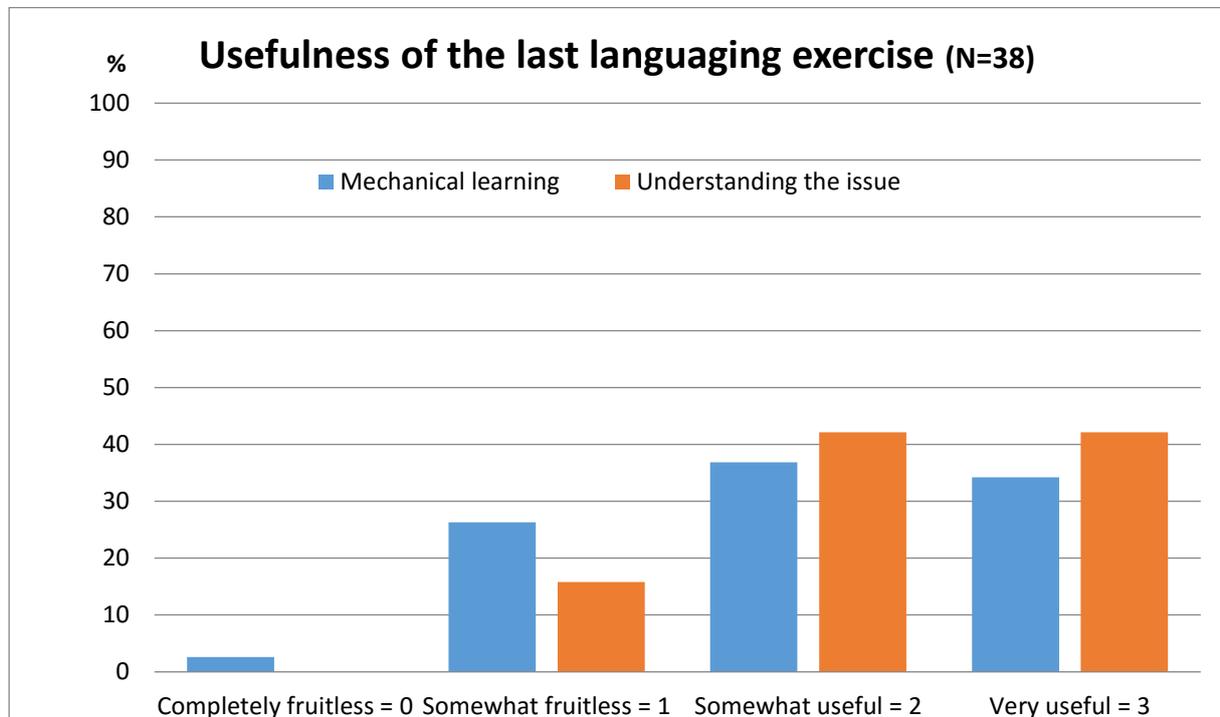


Fig. 9. Usefulness of the last languaging exercise on the course.

Figure 9 shows how useful the students found this last languaging exercise. Usefulness is divided into two parts as before. 24 students returned a solution to this exercise and 38 students answered to the question in the Figure 9. On the question “How useful it was to mechanical learning of the subject” average of the answers was 2,0 and the standard deviation was 0,8. On the question “How useful it was to understanding of the subject” average of the answers was 2,3 and the standard deviation was 0,7.

All of the languaging exercises on the course were thought an average to be useful both in mechanical learning and in understanding the issue, since all had an average over 1,5.

3.4 Students’ opinions on languaging expressed in open questions

There were four open questions regarding languaging in the questionnaire. All but one student answered the two general questions about languaging and around half answered the two specific questions about languaging exercise types and languaging exercises on the course. A few clear, often mentioned points arose from these answers. In general, languaging was thought to be a good and useful method which increases understanding. Many of the students were planning on using it in their coming assignments as it helps them not only to understand the issue better but also to revise easier. Some students also reported that they have already used languaging voluntarily even before this course i.e. written down verbal phases to clarify their solution. No one actually reported that there would be any harm in writing down more phases in the solution. Though it was thought to be useful it was also seen as a very arduous method. This is the main reason preventing many students from using it when

it's not required. Some students said that some of the exercises were so challenging that they were not able to even get started. Most of them thought that this is due to languaging being a new method and that they had not had any practise with easier exercises. It was seen that using languaging would be easier if it had started already on the easier courses. As for the feedback from the individual exercises or exercise types they were usually only single opinions pro or against. Only "seeking errors" -type got multiple comments, all of them against. Many of the students had also clarified that this type of exercise should be on a subject that is already familiar. On a new subject that you are just trying to learn this is far too difficult. This same result was seen also in Table 3 where the languaging exercise type 3 was clearly thought to be the least useful.

A number of students readily shared their feelings about languaging also face to face. The idea of languaging was thought to be a good and useful method also by these students but using the method was found to be difficult since it was new to everyone. Some specific types of exercises were thought to be useless in terms of understanding the concept better. The "seeking errors" -type was also in this context the least liked one.

3.5 Teacher's point of view

From the teacher's point of view languaging seems to be a very productive method looking at the results. Since over half of the students believed that their learning results had improved on this course because of the languaging exercises, it clearly seems to be a method worth using; especially since so many students thought that their understanding of the issues on the course had improved, which was the point of this experiment. Quite often improving one's understanding is a painstaking process and one has to work for it. That is why arduousness of exercises is not necessarily a bad thing from the teacher's point of view unless it leads to students not doing the exercises at all. Languaging can be a more laborious method for the teacher too. Making new exercises obviously means more work than using old ones especially if the types of the exercises are new also to the teacher. In addition, checking the students' solutions is often more time-consuming with languaging exercises.

3.6 Effect on grades

The grades on this implementation of the course were compared with the grades on the previous implementation. On both implementations the background information on the course was the same. The content, objective, extent and obligatoriness of the course were the same. Also the teacher in every lecture and exercise was the same. Only two things were changed on the course. Firstly, languaging was used in the exercises. Secondly, the fact that this study was conducted during the course and students were given extra points to their exam results if they participated in all of the study parts. Three students got a better grade because of these extra points but this factor is removed from the results. Of course, students' wish to get these points may have gotten them to study harder though getting these extra points did require only filling questionnaires and returning languaging exercises.

In 2014 29 students enrolled on an exam organised before the next implementation of the course. The average of the grades was 3,2 and the standard deviation was 1,7. In 2015 50 students enrolled on an exam organised before the next implementation of the course. The average of the grades was 3,4 and the standard deviation was 1,6. As always some students dropped out before the end of the course on both implementations and are left out from the results.

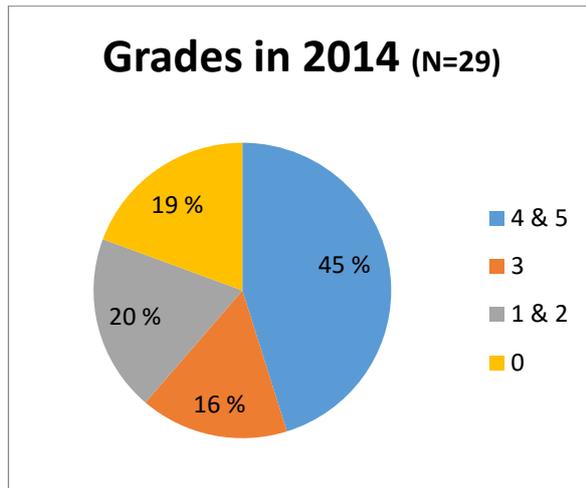


Fig. 10. Grades in 2014

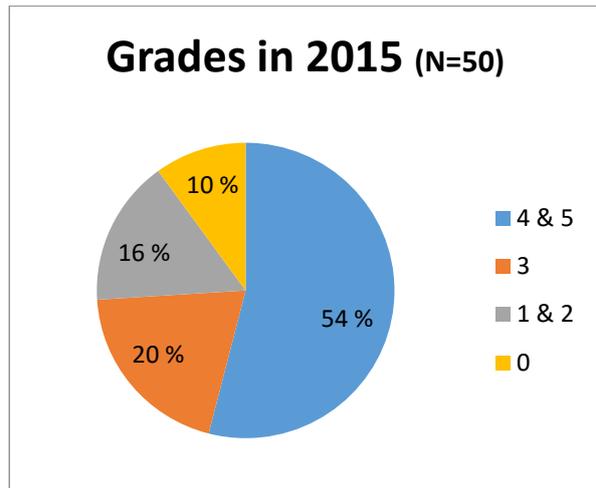


Fig. 11. Grades in 2015

As shown in Figures 10 and 11, the grades improved. The percentage of the best grades increased by 9 %. The percentage of the midmost passing grades increased by 4 %. The percentage of the worst passing grades decreased by 4 %. The percentage of the students failing the exams decreased by 9 %. The causes for this improvement can of course be numerous, but since nearly all factors in the course were same as the previous time, it is likely that large part of this improvement is due to use of languaging in the course.

4 CONCLUSIONS

Languaging is an effective method in building up students' understanding of the structural mechanics. This conclusion is supported by both students' own opinions and improved grades on the course.

More attention must be paid to making the exercises, especially the use of errors in a given solution. Error seeking will be beneficial only when the issue has already been learned.

Further studies will be carried out to get a wider sampling. The purpose is to make languaging a permanent part of every course in structural mechanics if the preliminary results are confirmed. The benefits will be greater if languaging is used systematically and starting from the first course. Then the method itself will already be known in the later courses and the students will have better understanding of the basic concepts of the field. Thus it will be easier to build further knowledge based on these concepts.

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